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Methodological aspects in integrating physical and psychological descriptions of human activity[1]

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Human activity was historically described assuming the viewpoint of physics and the viewpoint of psychology. In integrating these two descriptions, we meet both technical and methodological problems, and in this paper, we will mainly concern with the methodological aspects.

We need some preliminary choices to delimit the discussion, and a first choice entails the way of studying the facts with which we will deal. We decide that the facts investigated, and the procedure employed to study them, must be repeatable without any restrictions on principles or methods. We do not discuss here whether this requirement alone can characterize the scientific method. We only observe that it ensures the unlimited possibility of proving or disproving a fact, and this property is a frequently cited character of scientific method[3]. This repeatability requirement will prove to be a very strong methodological choice, whose consequences clearly delimit

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This report maintains the same approach. However, starting from its first version (July 2000), we adopted a different way of considering the constraints on the occurrence of the mental facts, and so we deeply reorganize the second part of the paper. In a further version (December 2000), we revised and added some consequences of this different viewpoint. In this version, we try to better characterize the framework of our discussion, and to clarify some aspects of mental fact description by adding a more extensive characterization of physical, psychic, and mental facts, and of the relationship between mental activity and mental facts.

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3. I think of Popper's falsification thesis. See K.R. Popper, *The logic of scientific discovery*, 1934, 1959.

our discussion[4]. The repeatability is a mental attitude, a way of considering the facts[5], but its consequences deeply influence the subsequent way of operating, as we will see in the following of the paper.

When we discuss the integration between physical and psychological description of human activity, we have to characterize both physical and mental facts and activities[6]. Since we inherit methodological problems from the philosophical tradition, of which the ontological dualism and the various kinds of reductionism are significant examples, we outline some issues that we will presuppose in our discussion.

In past papers[7], we can find a way of defining physical and psychic things by using mental activity. We still can interpret such a kind of definition as a description of our way to think of physical and psychic things. The leading idea was to define physical and psychic things as results of a perception, that we localize (in space for physical things, and in time for psychic things), and that we relate to other things of the same type. The relation, moreover, must be an action, and this choice means that we decide not to speak of physical or psychic things when we use, for instance, topological or order relation among the perceptual things. We prefer this choice because we can think of these situations as space or time structures. Their study is indeed a part of mathematics, although in mathematics we can use only things instead of perceptual things.

In the following of the paper, we will assume that, when we are dealing with psychic things, the actions can be either transitive or intransitive. In the first case we have an action of one thing onto another thing, or an interaction between two things; in the second case we relate the perceptive results at two moments to say, for instance, that something grows or diminishes. When we deal with physical things, we instead will assume that action is always transitive. As we will show later, physics inherits from elementary classical mechanics the use of a mental scheme in which the cause of a body movement is external to the body itself; so we will assume that the cause of any physical process is a physical thing different from the physical thing which changes. The conditions, that we require to hold in physics to apply a cause-effect relation, may be thus incompatible with the analogous conditions that we require to hold in psychology.

The previous characterization of physical and psychic things preserves many properties that our culture assigns to them. In particular, if we wish not to contradict ourselves, we must think of physical and psychic things as being subjects of actions and changes on other things of the same type, because this fact is implicit in their definition. Starting from this remark, we can maintain all the practical differences between physical and mental things and we can avoid

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4. I discussed this point in R. Beltrame, "On brain and mind", *Methodologia*, 10 (1992), pp. 7-13.

5. When, for instance, a celestial body approaches our sun, this mental attitude requires that we measure a certain number of parameters that concern the body, the sun, and the component of the solar system. We must measure the parameters that we think to be necessary for checking whether another celestial body will have the same behavior, when it approaches our sun with the same values of the measured parameters. The repeatability as mental attitude was discussed following this viewpoint in R. Beltrame, "Appunti di metodologia operativa: i caratteri costitutivi della scienza (Notes on operative methodology: the constitutive characters of science)", *Ricerche Metodologiche*, 3 (1968), pp. 23-40 (in Italian).

6. This point was discussed with more detail also in R.Beltrame, "Aspetti metodologici nella definizione dei fatti mentali edella loro dinamica (Methodological aspects of the mental facts definition, and of their dynamics)", in AA. VV:, *Categorie, Tempo e Linguaggio (Categories, Time, and Language)*, Quaderni di *Methodologia* 5, Roma 1998, pp. 45-100; and, in a previous version, as CNUCE Report C97-25, 1997 (both in Italian).

7. In the framework of the ideas of the Italian Operative School, a first characterization of physical, psychic and mental things is in S. Ceccato, *Un tecnico tra i filosofi (A technician among philosophers)*, Vol II, Padova 1966, at the section "Modificazioni ed innovazioni (Changes and upgrades) [1965]" pp. 27-30 (in Italian).

the ontological dualism between the two realms that we inherit from the history of philosophy. We recall that the ontological dualism was introduced because we accept that a physical thing, for instance fire, occurs as a cognitive fact only if we have the related cognitive activity, and we still accept that this cognitive activity will occur only when we have someone who does it. Nonetheless we equally accept that the fire burns a piece of wood and transforms it into ash, with no dependence on someone's thinking of these facts. That is, the occurrence of physical transformations can be neither forced, nor forbidden only by the mental activity of someone who thinks that they have or do not have to occur. It is not necessary to introduce two ontological different principles (one for the world of physical things, and the other for the world of mental facts) for explaining this difference after having thought of fire as being the subject of the burning activity and of the related consequences. We cannot ascribe it to another subject (for instance who is doing the mental activity of thinking that the fire burns a piece of wood) otherwise we would contradict ourselves. We decide not to contradict ourselves because we want to make inferences and logical deductions about physical facts whose occurrence has strong relevance because it may involve survival, and because from a contradiction we can deduce both a statement and its negatio. We thus require that our knowledge system will be free of contradictions at least in the parts that we use to do logical deductions about physical facts whose occurrence has strong relevance for our life.

In describing a way of thinking to physical and psychic things, we previously described mental facts through other mental facts. This procedure can be used to define a mental thing, but it cannot be used systematically. Starting from a certain point, we must define mental things in a different way; otherwise, we will have a *regressum ad infinitum*. Since we must satisfy the repeatability requirement that we stated above, we will assume that mental things are defined by referring to a physical description of the systems that we consider being able to perform the mental activity.

We require that the physical description deterministically predicts repeatable experiments: that is, it systematically uses the cause-effect relation instead of a mere correlation between the events, and a bijective function holds between the things that we relate as causes and their effects. According to this strictly deterministic viewpoint, an intuitive geometrical framework for many aspects of our discussion will be the dynamics of a physical system, which can be represented by trajectories that do not intersect in phase space. Although these assumptions are difficult to realize in practice[8], they become immediately evident the essential differences between a physical and a psychological description, and their methodological source. In fac6t, psychology classically defines its items in a different way because it has to admit the possibility that a mental fact or activity can occur again during the life of the same subject, and that they can occur to different subjects. From this way of defining mental things, it follows the theoretical possibility of their intersubjective character and so the theoretical possibility of communication between the subjects.

In this paper, we will assume that mental facts and activities are defined by an injective function into a subset of the physical processes that occur in the system that we consider to

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8. Difficulties mainly arise from the mass of information that we must involve, and from the essential nonlinearities of the theory. Clearly, we might trace back the differences between the two dynamics also when we use a more realistic physical description, for instance a description that follows the approach of statistical mechanics. Nonetheless, statistical mechanics would require a presentation that is more cumbersome, and that would mask essential differences between the physical and the psychological approach. However, the main reason is that I did not yet outline these differences with sufficient clarity in the framework of statistical mechanics, and of continuum mechanics.

be doing the mental activity. Since the physical processes are defined in a way that satisfies the repeatability requirement, this requirement is still satisfied in deciding the occurrence of a mental fact or activity. We also decided to use only a subset of the physical processes that we singled out to describe deterministically the dynamics of a physical system. We will show that a mental fact or activity can occur again during the life of the same subject, and that it can occur to different subjects. However, a further physical activity now accompanies the occurrence of a mental fact or activity, besides the process that we used to define it. We will show that we must go back to the physical description to predict the flow of the mental activity in a deterministic way, and that we can set only correlation between the occurrence of mental things.

The equations that describe the system evolution completely describe the dynamics of a physical system: that is, they describe the flow of the activity, the constraints on this flow, and the development of the system. We will show that we lose this syncretism, when we take the viewpoint of psychology. In particular, learning will appear only in this description, because in physical description architectural changes in the system are sufficient to fully describe its behavior. Constraints will lose the deterministic character that we usually assign to them, and they can predict the occurrence of the constrained fact only in probabilistic way. We shall conclude that dynamics cannot have the same description in the approach of physics and in the approach of psychology. Furthermore, we cannot map isomorphically one dynamics onto the other. We thus cannot assume a reductionistic position.

We will stress the role of the mental activity and of the conditions of its occurrence. We also will stress the importance that the acquired habits have in determining the humans' behavior. This idea is very old, and we can trace back it to Aristotle[9]. It emphasizes the importance of education, with the related social and personal responsibilities, particularly because our way of defining mental things excludes that we can derive a private character from their definition. So, we must obtain the same practical consequences by privacy: that is, by a legal statement, and social consent.

In this paper, we mainly have to deal with mental activity, and with the conditions of its occurrence. In some cases, a mental activity occurs, but we are only concerned with the mental thing that has this activity as constitutive, or with its consequences. As a rule, we will explicitly declare this change of viewpoint, but sometimes we will avoid to mention it explicitly, not to worsen the readability of the paper. We also assume that mathematical properties hold, which are necessary to have well defined the elements involved in the discussion. We may justify this decision because mathematical formalization of the ideas presented in this paper is out of the scope of the paper.

The characteristics of our reference physical description

The reader acquainted with physics can skip this section by assuming that we will refer to a physical description in which the dynamics of our system is geometrically represented by trajectories that do not intersect into a phase space of suitable dimensions. Since classical biological systems are opne to exchanges of matter and energy, we will assume that our physical description refers to a biological system and a suitable part of its environment such that the enlarged system can be considered as being isolated. We also assume interactions that we use

9. Aristotle, *Eth. Nic.*, II, **1**, 1103a 11 ff.

as elemental in the theory do not explicity depend on time. In the following of this section, we briefly will discuss these assumptions and we will show that they have a methodological character.

As we pointed out above, we assume in this paper that facts and procedure, with which we deal in our discussion, must be repeatable without restrictions on principles or methods.

In each experiment, for instance, we can have only one dependent variable, and we must study its dependence on only one independent variable. Then we must assign a constant value to all the other variables that we think may influence the experiments. These conditions are all necessary to have repeatable experiments. In mathematics, we have to deal with mental facts and their relations. Demonstrations take the role of the experiments in physics, we use explicit definitions to code mental facts and pieces of reasoning, and we introduce physical objects as symbols and rules for combining them. Then strings become suites of physical objects - pictures, drawings - and the demonstrations become equivalent to a sequence of rewriting strings. The rewriting procedure starts from the initial string and reaches the thesis string by using the specific hypotheses and the theorems that we previously proved: that is, it proves the equivalence of certain strings. The rewriting operation thus becomes a physical process by which one or more physical objects replace another physical object, and we use this fact to do arithmetic on machines: both the old mechanical ones, and the current electronic computers[10].

We do not accomplish the requirement that facts and procedure with which we deal in our study must be repeatable without restrictions on principles or methods, when the observation involves someone's account, description or testimony as a constitutive element: for instance, when the subject that we think to do the mental activity is a constitutive element. For the same reason, we cannot use accounts, descriptions, or testimonies of the persons, who we think as doing the mental activity, to identify the mental facts in experiments; and the testimony of the observer cannot be constitutive in an experiment[11]. More generally, we cannot claim to observe in a repeatable way the occurrence of things that we consider having a private character because this way of considering a thing implies limitations on the possibility of observing it, and so the two ways of considering a thing are not compatible[12]. In experiments, we thus cannot assume things having a private character as dependent variable, as independent variable, and as one of the parameters that characterize the experiment. In particular, we cannot give this role to mental things. Accounts, descriptions, and testimonies can only serve as indications to get back directly to what we want to assume either as the dependent variable, or as the independent variable, or as a parameter that characterizes the experiment. Although these constraints directly relate to the experiments, they also influence the theories, because a theory must give predictions that we can be test by such a kind of experiments to satisfy the previous repeatability requirement.

When we refer to the scheme in which we defined physical things by using mental activity, only the situations in which physical things are obtained from perception satisfy our repeatability requirement. We assume that this condition holds when we will refer to physical things in the following of the paper.

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10. Both arithmetics and the recent sophisticated computer programs of symbolic manipulation in algebra or in mathematical analysis are methodologically grounded on this viewpoint.

11. If the subject's testimony is constitutive, then we could not compare the results of experiments carried out on different subjects, and we should find the same limit if the testimony of the experimenter is constitutive.

12. The statement means that we cannot assume the consequences of the two categorizations to hold together.

Furthermore, physics inherits from elementary classical mechanics the use of a mental scheme in which the cause of a body movement is external to the body itself. Therefore, we cannot assume a mechanical body as causing its movement. We find it explicitly stated in Euler's *Mechanica*[13], and we must consider this assumption to be part of the definition of mechanical body when we are dealing with its dynamics. We still think of the action of a mechanical body on another mechanical body as a physical process, and this process implies a change in the agent too. The causes of this process must be external to the agent, to agree with the assumption stated above. Therefore, we can satisfy the previous requirements if we conceive of the action of a mechanical body on another mechanical body as an interaction, and if we assume that interaction is elemental in mechanics[14]. When the interaction is between two bodies, the previous choice implies two actions of equal intensity and opposite[15].

When in physics we represent elemental interactions by forces, this decision is equivalent to other strong methodological choices. Since we represent a force by a vector, and since vectors are defined on linear spaces, we can compose them linearly[16]. When we use a linear law of composition the result has always the same properties as the components, and we can safely transfer all the properties of the components to the result. Furthermore, the components are all independent, because in a linear composition a component that enters with zero weight does not modify the effect of the other components.

In classical non-relativistic mechanics, vectors are defined on spaces whose model is a three-dimensional Euclidean space. So, we have a finite orthonormal system, which is also a basis, and we have a scalar product with which we can compute the projections of a vector onto another vector[17]. Moreover, we also define as a vector the displacement of a mechanical body, which is the final effect of a force. We thus can define the scalar product of a force and the displacement of the point to which the force is applied. In this way, we obtain the value of the energy exchanged by the system as an effect of a displacement when a force is acting on the body.

In physics, we further assume that forces, which we use to describe elemental interactions, are conservative[18]. We recall that a force is said to be conservative when its work in moving a mechanical body does not depend on the trajectory along which the physical body is

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13. In Newton's formulation: "Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directu, nisi quatenus a viribus impressis cogitur statum illum mutare.", it is not sure whether the cause of a mechanical change must be external to the physical body. No doubt it is possible with regard to Euler's formulation: "Corpus absolute quiesciens perpetuo in quiete perseverare debet, nisi a causa externa ad motum sollicitetur." [L. Euler, *Mechanica sive motus scientia analytice exposita*, 1736, Ed. P. Stäckel. Leipzig, 1922, Vol. I, p. 27]. Feynman makes an analogous assumption by stating "that the force is equal to zero unless some physical body is present" [R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman lectures on Physics*, *cit*., Vol. I-1, pp.12.1 ff.]. In elementary mechanics we also think of the mechanical body as being atomic and not composed of parts; and this character must be considered part of the definition of mechanical body. Indeed, a single scalar and a direction, that is a single vector, completely describe the action of the environment on the body. Finally, when we think of the mechanical body as being composed of parts, this viewpoint is applied to the single parts of a body which we consider as being atomic (that is, the parts that we decided not to split again).

14. I think that we maintain this assumption in continuum mechanics as well, although its formulation is not so immediate.

15. The extension of this scheme by linearity to the case of N bodies is at the basis of the classical mechanics of systems.

16. The electrostatic action of N charged particles on one charged particle is a good example. It is a vector which is the sum of the N actions of each charged particle on the target one; although the single interaction is a non-linear function of the mutual distance between two charged particles.

17. We note that the projections of a vector onto the elements of the basis are also the components of a vector in the direct sum that represents the vector in the given basis.

18. See, for instance, R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman lectures on Physics*, *cit*., Vol. I-1, pp.14-8 and seq.

moved, but only on the starting and ending points of its trajectory. Conservative forces thus induce energy exchanges that do not depend on the particular process, and the quantity of energy exchanged is described by the differences of a scalar function: the potential. It is possible to prove that, when the force results from composition of conservative forces, it is conservative, and in this condition the total exchange of energy is the algebraic sum of the energy exchanges that depend on each potential.

Furthermore, the forces that we use as elemental forces must not have an explicit dependence on time[19]; and we extend this requirement to every interaction that we use as elemental in the theory. Moregenerally, this means that elemental interactions are independent of the observables that we use to distinguish the repetions of the experiment in which they are involved.

An explicit dependence on time for the interactions that we use as elemental in the theory would exclude the repeatability of the experiments. The requirement that conservative forces represent elemental interactions is equivalent to state that we can predict completely the energy exchanges between the system and its environment, and between the parts of the system. Therefore we can reverse the reasoning. We can assert that these requirements follow from the decision of having repeatable experiments, from the decision of predicting completely the energy exchanges between the parts of the system, and from the decision of predicting completely the energy exchanges between the system and its environment. The last decision is equivalent to state that we consider the system and its environment as being isolated, so that the total energy is constant in such enlarged system[20]. When we develop a theory, or when we apply it to a particular case, we thus have to enlarge the biological system to a part of its environment such that the biological system with this environment part can be considered as being an isolated system. We thus expect that some types of evolution may require particular characteristics of the environment as well.

However, only experiments can prove whether the properties mentioned above, and their consequences hold for a particular physical process. We have to prove whether we can pose a one-to-one relation between a vector and the values of the observables by which we manage the particular process. Then, we have to prove whether this vector has the properties of a conservative force. Finally, we have to prove whether this conservative force does not explicitly depend on time. When these properties hold, in the theory we can substitute the occurrence of that physical process with the action of a conservative force that does not explicitly depend on time[21].

In elementary Newtonian mechanics, we can geometrically depict the evolution of a system as a trajectory in a Euclidean space of six dimensions[22], the so-called phase space. As the system develops in time from an initial state, the image point traces a trajectory in phase

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[19.] On this point see, for instance, L.D. Landau, E.M. Lifshitz, *Course of Theoretical Physics*, Vol. I, *Mechanics*, 2nd Edition, London, 1969; and also W. Kölher, "Psychology and evolution", *Acta Psychologica*, 7, 1950. We recall that the basic relation of elementary Newtonian mechanics: $F=ma$, is invariant for reflection of the time coordinate: that is, by the change of the time coordinate t with $-t$. The reason is that acceleration is a second derivative with respect to time, and its sign does not change by changing t with $-t$. A very subtle discussion of the friction phenomena in relation with this point can be found in R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman lectures on Physics*, *cit*., Vol. I-1, 12-2.

[20.] Since we can neglect transformations between mass and energy in the biological system and in their environment, we can state that mass is also conserved in the enlarged system.

[21.] In Appendix A we will discuss some ways of weakening these requirements, and the related consequences.

[22.] We recall that in this context a system is always thought of as being atomic, that is we do not consider it as being composed of parts.

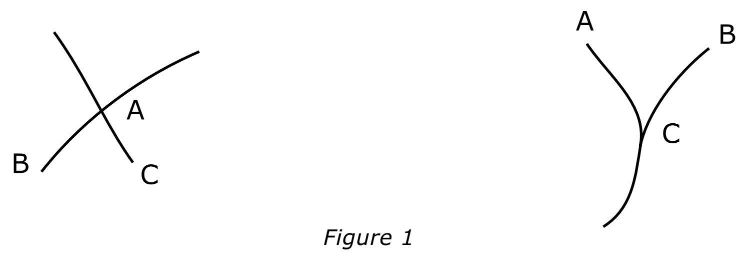

Figure 1

space. Therefore, a trajectory can be defined as a mapping of the real interval [0,1][23] into this phase space. The shape of the trajectories clearly depends on the interactions in the system[24].

We extend this representation to more complex systems by assigning a suitable number of dimensions to phase space[25], and the shape of the trajectories still depends on the interactions in the system. When the system satisfies the conditions stated above, the trajectories, which describe the possible evolution of the system, do not intersect[26]. In this case, the state of the system at one instant of time uniquely determines the state of the system later. We can reword this property by saying that each trajectory can be considered as being the effect of unique, specific set of physical facts. A bijection thus holds between this set of physical facts considered as being the cause, and the trajectory considered as being the related effect[27]. Therefore, this geometric representation depicts a strictly deterministic dynamics. Figure 1 shows the effects of losing the property that the trajectories do not intersect. The left hand of Figure 1 shows immediately that, starting from the state A, we can have two possible trajectories: the trajectory from A to B, and the trajectory from A to C; and we have an analogous situation when the trajectory has a loop. The conditions that determine A may thus predict either B or C as possible future states of the system. On the right, the different conditions that predict the two states A and B can both predict state C as possible future states of the system. In this second case, the trajectories do not violate the requirement stated above when we move top-down. However, they might introduce contradictions in the theory when we assume that conservative forces describe the elemental interactions, because the inverse processes do not satisfy the requirement that the related trajectories do not intersect in phase space, and conservative forces lead to reversible processes.

The geometrical representation of the dynamic of a system by trajectories that do not intersect in phase space thus means that we succeeded in individuating a suitable number of independent observables, and in confining the interactions, the non-linearity, and the non-locality of the theory into the description of the trajectories. Furthermore, since we assumed that our system is isolated, situations have no interest in which all the observables maintain

23. We can identify this interval with an arbitrary interval of time because we can assume a suitable scale factor, and we are interested only in finite intervals of time.

24. We implicitly assume that the interactions are described by a potential which has no explicit dependence on time; otherwise, as we have seen, we may lose the repeatability of the experiments.

25. The phase space has a number of dimensions that is the double of the degree of freedom of the system.

26. A very clear and compact treatment of these topics can be found in J.L. Singe, *Classical Dynamics*, Encyclopedia of Physics, Vol. III/1, Springer-Verlag, New York, 1960, pp. 98 ff.

27. For the sake of simplicity, we can cite the inertial motions of a mechanical system. For instance, the trajectories of inertial motions with the same momentum are parallel straight lines in the three-dimensional subspace of phase space which identifies the spatial coordinates. To these lines we have to add the same point in the subspace of phase space which identifies the momentum coordinates, and we obtain the possible future trajectories of such a mechanical system. Clearly the trajectories do not intersect.

the same value over a finite interval of time, because an isolated system shall maintain that state since it remains isolated. Therefore, we will assume that a line in phase space always represents the dynamics of our systems, and we are no longer interested in a trajectory when it reduces to a point. Since our discussion has a methodological character, we will refer to a representation of the dynamics in which trajectories have no intersection[28]. We outline that the deterministic character refers to the predictions that are allowed by the theory. Although trajectories that do not intersect in phase space describe the dynamics of a system, this property does not necessary hold when we consider projections of trajectory segments onto subspaces of phase space. This remark will be crucial in discussing the integration of the two approaches.

The conditions discussed above are rather severe, and we expect difficulties to satisfy them in biology. As we know from the physics of complex systems, we do not find methodological obstacles to imagining a theory of the behavior of biological systems in which all the intermediate explanatory elements are physical processes that occur in the biological system, and in its environment. We neither find methodological obstacles in imagining that we can describe these physical processes according to the requirements discussed above. The practical difficulties are a different thing, because systems that are studied by biology usually cannot maintain the architecture and the activity on which we are interested without exchanging matter and energy with their environment. This means that, without these exchanges, these systems lose the properties by which we study them, and very frequently, they disassemble. For this reason, biology frequently assumes to study open systems. Clearly, this statement does not fit with our previous assumptions. In our methodological discussion, we thus will not assume that biological systems are studied as open systems, although we are fully aware of the practical difficulties that are implicit in developing a theory in which we must include the environment to obtain a satisfactory dynamics of a biological system.

This strategy has today severe limits when we try to apply it in biology, because we usually do not know with sufficient detail the quantitative aspects of the energy exchanges in biological processes, and we always have poor knowledge of the parts of the enlarged system[29] that are involved in these energy exchanges. Moreover, when we make *in vitro* experiments, conditions of the experiment, that are often equivalent to postulating practically unlimited sources and sinks of energy, may mask these problems. Therefore, in biology the energy balance equations do not play the essential role that they have in physics. When we deal with systems that we consider as doing mental activity, we meet practical difficulties because the part of their environment may be large, that allows us to consider the enlarged systems as being isolated.

Experiments lead to analogous problems, but a well-assessed strategy was devised to manage the difficulties. Since scientific praxis requires that experiments be repeatable, in each experiment we must study how a single variable depends on another single variable after having set the value of a certain number of other observables. The values of these observables characterize the conditions in which the experiment is done, and we carefully must reproduce them to repeat the experiment correctly. When we can consider our system as an isolated system, all the observables belong to the system. When the system is not an isolated system, the

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28. Matematically, this property is equivalent to the statement that the inverse map of a trajectory into the real interval [0.1] is a function.

29. We refer here to the biological system and a suitable part of its environment so that we can consider the enlarged system as being isolated.

observables that characterize the conditions of the experiment still may belong to the environment. The strategy assumes that a surface separates the system from its environment, and in laboratory experiments, the system is always a bounded region of a three-dimensional space. We then substitute the value of the environment observables with the values of suitable observables on this surface, and we use these values to describe the interaction between the system and the environment at a given instant of time.

Furthermore, a single experiment studies a particular aspect of the system's behavior: the relation between the dependent and the independent variables, having fixed certain conditions. The theory then integrates the results related to different conditions. If our interest is only in steady states of the system, then we obtain a first level of integration by planning a series of experiments in which we impose different values on the independent variable, and in which the control variables that characterize the experiment have the same fixed values. In this way we obtain a relation between the dependent and the independent variables of the experiments, and this relation holds when the control variables have the fixed values assigned to them in the series of experiments. The theory usually requires several of these series of experiments to describe how the relation between the dependent and the independent variable depends on the control variables of the experiments. Moreover, the theory requires experiments in which independent and dependent variables describe interactions with the environment, because biological systems are usually open to exchanges of matter and energy with their environment. Therefore we must use the strategy described above only as a tool to simplify the management of the experiments, but we shall assume that a satisfactory theory must involve isolated systems as we discussed above.

When the analysis of steady states does not give a satisfactory description of the system's behavior, we have a higher order of complexity because in each experiment we have conceptually to substitute the single value of the observables with a function of time[30]. This substitution raises methodological problems, because we cannot violate, even implicitly, the requirement that the experiments are repeatable. Furthermore, when a system is in a steady state, we can refer to the same state the value of an observable irrespective of the duration of the measurement; and all the measurements of the observables refer to the same state of the system, even if a certain interval of time separates two measurements[31]. Both these very convenient properties do not hold when the system is not in a steady state. The measurement techniques thus become more difficult, and in the theory, we have to decide how to relate the result of a measurement with the value of the observable that we introduced in the theory, or that we planned to measure[32].

Other practical difficulties arise from the number of elements involved: that is, from the number of dimensions of the phase space, and so from the bulk of information that we have to know. A reasonable estimate is that the human brain contains about 10^{11} neurons. This figure alone should force us to apply the approach of statistical mechanics. Moreover, we must consider each cell as being a complex system, and we must account for a considerably greater

30. More generally, time here means an observable whose values have the mathematical properties of a totally ordered set.

31. This property is particularly useful when we have to determine the values of a function derivatives. Recall that the derivative of a function is a continuous linear operator at every point in which it exists. For real functions defined on a real space having finite dimensions, we thus need an array of values to characterize its derivative at a given point.

32. We have to decide, for instance, whether we will use the measured value as the value of the observable at a certain instant of time, or as the average value over a certain interval of time. This point is discussed in great detail in W. Grandy Jr., *Foundations of Statistical Mechanics*, *cit,* particularly at the beginning of the Vol. 2, *Nonequilibrium Phenomena*.

number of elements to obtain a physical description with the characters outlined above. However, the most severe difficult arises because the interactions between the elements of our theory are typically non-local and non-linear, as we will show in the next sections. In statistical mechanics, free-particle models are somewhat simple, although we have to deal with a number of particles that is in the range of Avogadro's number: that is, in the order of $6\ 10^{23}$ particles per mole[33]. Despite the difficulties mentioned above, the characters that we proposed for the physical description have a very high conceptual and methodological importance. They characterize a reference theory that will be a good instrument to clarify the foundations of the physical and psychological descriptions of the behavior of our systems. Today, we cannot realize a physical theory with these characters, due to the practical difficulty of obtaining and managing sufficiently detailed information about the interaction between the parts of a biological system, and between the biological system and its environment. However, when we are interested in a limited volume of phase space, and in a limited interval of time, the picture described above also can have practical relevance.

In the next sections we will see that in psychology we develop a theory that has different characters, mainly because we decided to define mental things by using only a part of the physical processes that we must introduce to have a physical description that satisfies the previous requirements. This decision relates to the requirement that the defined things can occur again during the life of the same subject, and that they can occur to different subjects. However, we automatically obtain a dynamics in which a bijective function does not hold between the things that we use as cause and the things that we use as the related effect. Moreover, this conclusion holds for every thing is defined by using only a part of the physical processes that we must introduce to have a deterministic theory of the physical system activity. It thus holds, for instance, for a movement of an animal, or for a part of the cell activity like DNA transcription. The dynamics of this type of things does not have the properties of a dynamics that can be geometrically represented by trajectories that do not intersect in phase space. It has instead the properties of the trajectory projections onto subspaces of phase space.

Defining mental things

We recall that we decided that the following conditions must hold in our discussion on integrating physical and psychological description of human activity:

- the investigated facts, and the procedure employed to study the facts must be repeatable without any restriction on principle or method;
- we require the theoretical possibility that a mental fact or activity can occur again during the life of the same subject, and that it can occur to different subjects.

We cannot claim to observe in a repeatable way the occurrence of things that we consider as having a private character, because the observation should involve someone's account, description or testimony as a constitutive element: in our case the subject that we think to do the mental activity. In particular, only the situations in which physical things are all obtained from perception satisfy our repeatability requirement. Therefore, when we decide to satisfy this requirement in studying humans, we cannot use accounts, descriptions, or testimonies of the persons, who we think as doing the mental activity, to identify the mental facts. For the same reason the testimony of the observer cannot be constitutive of an experiment[34]. In sci-

[33] See, for instance, W. Grandy Jr., *Foundations of Statistical Mechanics*, Vol. 1, *Equilibrium Theory*, *cit.*, Chap. 5.

entific experiments the dependent variable, the independent variable, and the parameters that characterize a scientific experiment can only be physical facts that relate to the body of the person that we assume as doing mental activity, or physical transformations that he does on other objects, and these physical facts must be obtained from perception. Although these constraints directly relate to the experiments, they also influence the theories, because in scientific activity a theory must give predictions that can be tested by scientific experiments. Furthermore, if we consider the repeatability requirement as a character of the scientific method, then we must define mental facts and activities by using physical facts, before introducing them in any scientific context.

We can satisfy both the requirements stated at the beginning of this section when we assume that mental facts are defined through a mapping into physical things. In the following of the discussion, we thus will assume that definitions use physical processes that occur in the physical system that we assume to be doing mental activity. We further characterize the mapping in the following way. We decide that a mental fact or activity occurs every time the physical process occurs that we used for defining it, and we still decide that, whenever we predict the occurrence of a mental fact or activity, we also predict the occurrence of the physical process that we used to define it. Finally, when the physical process does not occur, that we used to define a mental fact or activity, we decide that the related mental fact or activity did not occur either. In this way, we can assert without ambiguity whether a mental fact occurs, and we can test its occurrence by repeatable experiments on the systems that we plan to use in the experiments.

In mathematics, it is usual to characterize the mapping that we proposed above, as an injective function $f : (M \rightarrow P)$ of the set M of the mental facts and activities that we have to define, into a subset P of the physical facts occurring in the systems that we plan to use in the experiments[35]. Clearly, we cannot introduce hypothetical physical facts in the definitions, and we cannot substitute a physical process with a mental category or more generally with another mental fact or activity, because we would really define a mental thing through other mental things[36]. We thus obtain an unsatisfactory definition when, for instance, we use something that is described only as a change of state in a physical system[37].

The methodologically simplest choice seems to use all the physical processes that were necessary to develop a deterministic dynamics of the physical systems that we think of as being able to do mental activity. However, we can immediately note that mental facts and activities so defined might not occur again during the life of the same subject or to subjects

34. If the subject testimony is constitutive, then we could not compare the results of experiments carried out on different subjects, and we should find the same limit if the testimony of the experimenter is constitutive.

35. We recall some mathematical definitions that we use here. A map is defined by a triple (G, X, Y) where X and Y are sets, and $G \subseteq X \times Y$; the set G is said to be the graph of the map. A map will be called a function when it is single-valued: that is, when it assigns to each element $x \in X$ exactly one element $y \in Y$ such that $(x, y) \in G$. The functions will be notated $f :(X \rightarrow Y)$ and $y = f(x)$. The set X is called the domain of the function f, and the subset $f(X) \subseteq Y$ its range. When $x \neq y$ implies $f(x) \neq f(y)$ the function is said to be injective. When $f(X) = Y$ the function is said to be surjective (onto). A function is said to be bijective when it is both injective and surjective. A function f is said to be left invertible when there exists a function $g : (Y \rightarrow X)$ such that gf is the identity function on the set X. A function f is said to be right invertible when there exists a function $g : (Y \rightarrow X)$ such that fg is the identity function on the set Y, and a function is said to be invertible when it is both left invertible and right invertible; we can prove that an injective function is left invertible, and that a bijective function is invertible.

36. This remark also holds when we consider more general levels of a theory. At these levels of a theory we must use mental categories to obtain the required generality, but, if we wish to start from one of these levels and to use a top-down approach, then we must develop the theory and introduce the necessary definitions until we reach the level of specificity that ensures the link with repeatable experiments.

having a different history. Furthermore, this solution would lead us to define more mental facts than those we use in our cultures. We thus decide to use only a part of the physical processes that were necessary to describe deterministically the dynamics of our systems, are used to define mental facts or activities. Formally, this choice means that the injective function into physical processes, which we will use in defining mental facts or activities, is not also surjective; thus, it is only left invertible. Many physical processes can share the part of the processes that we used for defining mental things, and we will obtain a dynamics in which the occurrence of a mental fact or activity cannot be deterministically predicted by using only the occurrence of other mental facts.

The choice of the physical processes to use in defining mental things is a critical decision, as in any theory. We have to decide how much our definitions will be independent from the peculiarities of the physical architecture of the system. This architecture depends on the history of the biological system and of its interactions with the environment, because memory phenomena typically occur in biological systems. We can give the best solution to these problems when we are dealing with specific situations. However, we must exclude physical processes that would force us to assume that mental activity occurs continuously. Furthermore, when in physical description of the behavior of a biological system we find that the system is stable in a range of values of certain observables, we can use changes in the value of these observables to define mental facts and activities: for instance, quantitative changes in metabolic activity, or in molecule exchanges between cells and their intercellular space[38]. We thus have a great range of possibilities in defining mental activities.

We are completely free of choosing the mental and the physical facts that we connect when we define mental facts, because we are dealing with definitions. We instead have no margin of freedom when we are dealing with physical facts that shall follow from other physical facts, because we decided that observation and experiment would be determinant in this case. In particular, we showed that interaction between remote parts is necessary to develop a theory of the physical activity in biological systems, that this theory is typically non-linear, and that memory phenomena play an essential role. Therefore, many points of this paper will have the character of a discussion about the possible mental activity because of our freedom in choosing how to define mental activity.

We decided to define mental facts or activities by using physical processes, but we might decide to define mental states through a mapping of physical states onto mental states. If this mapping will be necessary, it must satisfy the same properties as the analogous mapping that we proposed for defining mental facts or activities: that is, we must require that an injective function holds of mental states into the physical states, and that a one-to-one function holds between their occurrence. In physical description, however, we prefer to consider as a particular type of process the situation in which the values of the observables that characterize the

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[37.] This point was not sufficiently emphasized in my past papers: for instance, R. Beltrame, "La première machine sémantique", *4me Congres International de Cybernetique*, Namur, 1964; R. Beltrame, "L'analisi in operazioni (the analysis in operations)", *Nuovo 75*, 1 (1967), pp. 17-21 (in Italian); R. Beltrame, "Osservazione e descrizione meccaniche (Mechanization of observation and description)", in *Corso di Linguistica Operativa (Course of Operative Linguistics)*, S. Ceccato Ed., Milano, 1969, pp. 115-139 (in Italian); R. Beltrame, "Perceptive Operations", *Thought and Language in operations*, I, 2 (1970), pp. 174-198; R. Beltrame, "Methodological aspects of a theory of the mental activity", *Methodologia*, 7 (1990), pp. 53-84 (in Italian).

[38.] When we mention the use of physical processes to define mental facts and activities, we shall think of the physical processes as having this wide meaning, and this meaning is in good agreement with the viewpoint of physics, where changes in the value of some observable are a way of defining a physical process.

process do not change during a certain interval of time. Moreover, we will not use a state of the physical system as cause, but we instead will use as cause the process that brought the system to that state. With these assumptions, we can use only processes in our theory.

We have to clarify why we use a mapping into physical processes for defining mental things, instead of using identification. By using a mapping, we wish to avoid that the two things have the same description when we look at them as mental facts. Identification, instead, might imply that the two things have the same description. This approach is compatible with the following strategy. We refer to this mapping when we use mental activity as a description tool. We thus have a description of what we use as tool and a different description of what we look as object of our description. In particular, we are only concerned with the occurrence of what we use as tool, and we thus avoid going back from the same type of description to the same type of description in a *regressum ad infinitum*. When we instead describe a mental thing, we can continue to use the injective function into physical processes, but we can also describe it by means of other mental things. In this last case, however, we must use a mapping into physical processes for defining the mental things that we assume as being elemental in our description; otherwise, we still will have a *regressum ad infinitum*.

Moreover, the choice of an injective function instead of identification allows us to maintain the greatest number of properties that our culture attaches to mental things. When we deal with physical things, we use perception, space localization and a relation with a thing of the same type. Identification of these things with mental things will not succeed in fully covering what our culture considers a mental thing.

As we will discuss in the following of the paper, this way of defining mental things does not agree with the conditions that we require for considering mental things as having a private character. We thus shall resort to legal constraints for achieving through privacy the same practical consequences that we expected from things that we considered having a private character as consequence of their definition.

Mental facts and mental activity

Until now, we used the two phrases ‘mental facts’ and ‘mental activity’ without specifying their differences. They reflect two main schemes that historically were employed to think about mental things. The more common scheme thinks of mental things as entities, and the word ‘mind’ designates the collection of these entities. When we study the occurrence of these entities, we must introduce a specific activity to speak properly of their occurrence. Usually, this activity is simply ascribed, as a faculty, to the subjects that we consider as being able to do mental activity. The second scheme instead conceives mental things as results of an activity. The word ‘mind’ now designates the subject of this activity, the activity is qualified mental activity, and, when we relate this activity to the results, it is qualified as being constitutive of the mental things[39].

In the following of the paper, we will relate the two orders of things by thinking of mental activity as being constitutive of the entities of the first scheme, and, as we have seen, we use the second scheme discussed above for defining mental things. That is, we prefer to define

39. This strategy was followed by the Italian Operative School in developing a model of mental activity, A good description of this model can be found in S. Ceccato, “A Model of the Mind”, in E. Caianiello Ed., *Cybernetics of Neural Processes*, Quaderni della Ricerca Scientifica, CNR Roma, 1965, pp. 21-79. A clear sketch of the history of the Italian Operative School can be found in V. Somenzi, “The Italian operative school”, *Methodologia*, 1, 1987, pp. 59-66.

mental activities by an injective function into a part of the physical processes that occur in the system that we consider as doing mental activity. We then think of mental facts as clusters of activities, and these activities become constitutive of mental facts. Furthermore, we will use the sentences, 'mental facts' and 'mental activities', with the meaning discussed above. We will use 'mental things' when our statement applies to both mental facts and mental activities.

Mental facts can be different in different individuals, and they can have a different stability in the same individual through his life. This approach eases us in integrating our definition of mental activity with the physical description that we decided to assume as reference, because we have to map an activity onto another activity. Unfortunately, as we shall see below, no isomorphism holds between the physical and the psychological theory of the systems' behavior. No reductionism is thus possible.

If we would use the first scheme, which considers mental things as entities, we will have a more direct connection with culture: that is with the set of elements that are transmitted to individuals by the group in which they live. The reason is that this scheme directly deals with mental facts. We are however at a disadvantage in building a theory with a satisfactory degree of generality because the choice of mental facts escapes with difficulty the influence of a particular cultural context. Therefore, when we develop a theory, the habits, that are active at a particular historical moment in the group we are studying, may easily mask the possibilities of the biological architecture. A general theory instead considers these habits as variables, because they explain and predict some behavior differences between individuals, and between different moments in the life of an individual.

Moreover, learning is a continuous source of new mental entities, and linguistic communication, both spoken and written, is a great source of training in humans. If we assume that all mental entities are atomic, then we can introduce new mental entities into the theory only by new definitions; and a theory of this type becomes unmanageable. We might try to circumvent this difficulty by developing a theory in which mental entities will be composed of a few other mental entities. However, we know that only exceptionally an entity will be decomposed into a pure sequence of more simple ones. We typically have decomposition into more simple mental entities and relations among them. These relationships, however, are often a by-product of the decomposition criteria. They are thus part of the mental activity of the observer, and we cannot consider them as being part of the mental activity of the observed subject. We meet difficulties to integrate this approach with a physical description of the systems that we consider as doing mental activity.

In a representation of the dynamics of our systems as trajectories that do not intersect in phase space, mental things are defined by projections, onto subspaces of phase space, of one or more segments of the trajectory that describe the evolution of the system in phase space. Note that projections can have many configurations: for instance, they can be projections of the same segment onto different subspaces of phase space, or they can be projections of successive sub-segments onto the same or different subspaces, and we implicitly define the timing pattern of their occurrence as well. We thus have many possibilities when we define a thing in this way. The simplest one consists in defining the occurrence of a mental thing by the occurrence of a process whose geometrical representation is a segment of line into a subspace of phase space.

Since many trajectory segments can share the same projection onto a subspace of phase space, the occurrence of what we defined by a projection can be obtained by doing the physical activity that we described by the trajectory segments that share the projection[40]. We can thus realize the same mental thing in different contexts of activity, and by a different biological architecture. In particular, the same individual can do it in different moments of his life, and different individuals of the same biological species or individuals of different biological species can do it as well. We thus satisfy the requirements mentioned above, and we also escape the necessity of defining anthropomorphically the mental activity.

If we prefer to avoid this geometrical representation, we can simply reword the previous picture in the following way. The trajectories in phase space become the description of how the physical system evolves from certain initial conditions, and the equations that describe this evolution fully describe the dynamics of the system. The trajectory segments in phase space become the physical processes that we introduced in our theory to explain the behavior of our systems so that cause-effect relations will be one-to-one. Their projections become the sub-processes of the physical processes that we used for defining mental things, or other elements of psychological theory, in such a way that they can occur again in the life of the same subject, and in different subjects.

However, constitutive activity usually does not interest the subjects who are doing mental activity, because they are mainly concerned with relations among the things that the analyst considers as being results of constitutive mental activity. Our languages reflect these alternatives, because they have an equal possibility to emphasize a relation among things, or the mental activity by which someone sets a relation among things. We can say, for instance, 'the cat was near the door', or 'I saw the cat near the door', or 'I am thinking that the cat was near the door', and so on[41]. When we concern ourselves with relation among things, we deal with things that we can continue to describe through their constitutive mental activity, but we enter the realm of complex mental facts[42]. This fact would favor an alternative strategy of defining mental things, which starts from the current knowledge system, from the learning rules, and from the knowledge of the interactions with the environment.

The two strategies have opposite approaches because one refers to mental activity and the other deals with mental facts, and they are not equally reliable. The first strategy really delimits the possibilities because it is grounded on a deterministic dynamics of the physical system, and it allows for an unlimited number of mental facts. The second strategy meets difficulties to account for the possible new definitions of mental things, but it is more suitable to account for the development of which subjects are aware, and so it is very interesting in humanities.

In this paper, we systematically will follow the first strategy. The possibility of a mental fact thus becomes conceptually linked to the possibility of its realization into the dynamics of the classical biological system and a suitable part of its environment, such that we can consider the enlarged system as being isolated. This high degree of freedom is reduced by the particular evolution in the case of a single individual, and in the geometrical picture that we systematically used along the paper; this reduction corresponds to pass from the system of trajectories to a single trajectory, but the possibilities still remain very high.

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40. This fact shows a further source of the effects that we usually ascribe to the plasticity of the nervous system, and this source does not require the local changes in the biological system architecture, that we mentioned in discussing memory phenomena. In particular, we are not required to introduce new learning activity.

41. We have no *regressum ad infinitum*. We only have to describe the mental activity by which we obtain the different situations that we can think of, and the mental activity by which we consider a thing as being a mental thing.

42. We outlined few situations of this type when we discussed how to think of physical and psychic things in terms of mental activity, and when we showed that many usual properties follow from a requirement not to have contradictions.

Further remarks on defining mental things

The oldest and steadiest way to realize a mapping between physical and psychological facts is a relation between functions and organs, where functions pertain to the psychological description, and organs pertain to a physical description. This strategy has a certain number of disadvantages, mainly because the relation organ-function is useful only when the organ is specific: that is, when it is related to a single function, and it is a well-delimited anatomical piece. Many situations cannot satisfy these requirements: typically, for instance, the colors whose characters can vary continuously. In fact, an extremely high number of organs should be necessary. We meet an analogous difficulty when we think of the function as being realized by an integrated activity of different parts, and the integrated activity depends on external factors. A good example is given by a cat that coordinates its movements to land on its paws after it has fallen off a wall. The movements and their coordination hardly are ever identical, because they depend on the initial conditions of the fall, and clearly we cannot suppose that each coordination have its proper organ, because the number of possible different situations is extremely high, and the organ-function relation should lead to an unlikely number of organs.

The relation function-functioning is suitable for defining mental facts when we can assume that a function is realized in only one way, that is by means of a unique physical process. In this case, however, we can use a direct mapping into this process, and we relapse into the previous way of defining mental things. The use of a function-functioning relation may become misleading because, when we realize a function, we can think that we also attain an aim. In this case, we have as constraint on our mental activity that the activities by which we can accomplish the same aim will be considered as being equivalent and interchangeable to attain the aim. We thus lose the injective mapping that is necessary to have a suitable definition[43]. Furthermore, the idea of function is related to the idea of purpose. In scientific theories, however, we prefer to introduce only material or efficient causes, because we can immediately plan an experiment, in which we start the process that in the theory is considered the material cause and we look for the expected effect. If we will instead use a final cause, then we fall into a rather intriguing situation, because a final cause is thought to be at the end of the process that it causes, and so we must introduce another thing that promotes the expected result. In conclusion, either we would lose the one-to-one function into physical things that we required to our definitions of mental things, or we shall complicate the theory in an unacceptable way, because we must duplicate the elements that we think to promote the occurrence of the facts.

Furthermore, in psychology subject has often the role of acting cause in passing from purpose to action. When the biological organism acting as subject has a too poor complexity, other explanatory elements were introduced, and instinct is the most common one. However, biology today has no necessity to introduce in its explanations purposeful elements, even survival. We can use a mental scheme in which we associate a different life expectation of the organism to the biological processes that occur in it. When we sample the organisms in our observations, the life expectation associated to a biological process becomes a monotone function of the probability to find an organism in which that process occurred. Therefore, we do not require any purposeful element to obtain a satisfactory theory of evolution, and a satisfactory explanation for the observed distribution of the various species[44]. In my opinion, because of this change of perspective, purposeful behaviors survive now in psychology only among the

43. Since the life expectation of a biological organism usually increases when the organism can attain a certain result by using more than one process, we might think of having proposed a useful definition.

highly complex behaviors, and we find another good argument to avoid organ-function or function-functioning relation when we define mental things.

In past papers, I tried to define mental things introducing the assumption that they have as counterpart physical processes that have a common part[45], and this idea can be traced back to the notion of selective attention in W. James' *The Principles of Psychology*[46]. The picture that we outlined in the previous section allows us to give the following meaning to this way of defining mental things. The reference physical description, as we have seen, requires a certain number of processes, and we decided to use only a part of these processes to define mental things. When the dynamic of our systems can be represented by trajectories that do not intersect in phase space, the physical processes that we use to define the mental things are represented by projections of segments of the trajectories that describe the system dynamics, and each projection is onto a suitable subspace of phase space. Let $\{S_i\}$ the set of these subspaces. If we define mental things with physical processes that have a common process as component, two conditions must be satisfied. These projections must have a common projection onto a subspace of phase space that must be a nonempty intersection of the S_i. The subspace that is the intersection of the S_i must be orthogonal to the subspaces of all the projection that are not used to define mental things[47]. We probably may satisfy these requirements in defining mental things, because phase space has a great number of dimensions, and we can expect to find a projection that is shared by a great number of trajectory segments. However, the condition of having as counterpart a common physical process does not follow from the assumptions made at the beginning of the paper: it is a further property that we can require to our definitions of the mental things.

A statement that we cannot attend to more than one mental thing together is only compatible with a decomposition of the mental facts or activities into a chain of other mental facts or activities. It is not compatible with decomposition into a net. The physical description does not have this restriction, and we meet one of the reasons that prevent an isomorphism between physical and psychological description of the behavior. When we define mental things through an injective function into physical processes, we can get around this limitation by considering a suitable subset of a net of physical processes as being one process. However, this trick fully works only when we define mental things that we consider as being atomic, otherwise, we must carefully check against possible incompatibilities.

We can use mental things to define a new mental thing, but only if we previously defined them by an injective function into physical things. Since this use becomes simply a substitution of names, it becomes a shortcut of no theoretical interest in a methodological discussion, although it can be useful in communicating our definitions. In general, the new mental things result from a non-linear law of composition, and we can describe this non-linearity by saying that we compose things that are thought to have qualitative differences. The properties of the

44. See for instance L.L. Cavalli-Sforza, M.W. Feldman, *Cultural transmission and evolution: a quantitative approach*, Princeton, 1981.

45. See R. Beltrame, "Perceptive Operations", *cit.*, pp. 179-183; and the review of the past model of the mind in R. Beltrame, "Methodological aspects of a theory of the mental activity", *cit.,* (in Italian). In these papers I did not stress sufficiently that mental things are to be defined. So, this character may be also thought as a property of the mental things, instead of a requirement of their definition. If we decide to use a different strategy for defining mental things, then this character has to be proved by means of experiments, because it relates to physical facts.

46. See W. James, *The Principles of Psychology*, 1890, Dover, New York 1950, Vol. I, pp. 402 ff.

47. This condition ensures that the characterizing process belongs only to the processes that are used to define mental things.

result are thus different from the properties of the components, and we must investigate the properties of each mental thing that we defined in this way.

We can assert that the occurrence of a mental thing implies the occurrence of its constitutive activity. This assertion becomes a truism when we assume that the injective function into physical processes, by which we define a mental thing, also defines its constitutive activity[48]. However, subjects are rarely interested in constitutive activity of mental facts. This interest is typical of a well-delimited chapter of psychology, and the more frequent interest is in relations among things. Moreover, subjects are frequently interested in relations among physical and psychic things. In this framework, subjects usually think of mental facts as facts whose occurrence induces the occurrence of other facts, frequently of physical facts: for instance, the utterance of a word or phrase is thought to follow the occurrence of a mental activity. This way of thinking introduces the further requirement that the occurrence of a mental fact shall be followed by the occurrence of a physical process. The framework that we previously outlined eliminates the difficulties of the classical ontological dualism. When we refer to a geometrical representation of the dynamics of our physical systems as trajectories that do not intersect in phase space, the previous requirement acquires the following general representation. The counterpart of linguistic activity is a correlation between projections of trajectory segments: one projection represents the physical process that we used to define the thing that we assume to be the designated thing, the other represents the physical process that we used to define the thing that we assume as designation[49]. Clearly, the consequences follow of relating things that we defined as projections, onto subspaces of phase space, of trajectory segments in phase space. As we have seen, many trajectory segments can share the same projection, and we do not succeed in connecting deterministically the occurrence of things that we defined as above.

As immediate corollary, we cannot use language to define mental things according to the requirements that we stated at the beginning of the paper. Two historical facts confirm this remark. Certain sounds and graphic material are assumed to designate the same mental fact at the interior of the same language, and the equivalence is here set at level of designating things. Facing to the clear differences in the designating material of different languages, we meet the assumption that the same mental thing underlies different languages. Aristotle explicitly stated this assumption in his *De Interpretatione*[50]; and the weaker assumption that a common mental activity underlies different languages does not face to the difficulties. Furthermore, subjects can use spoken or written language to designate the same mental fact, and they can use one language of those that they know. We thus lose again the injective function that is necessary for a definition.

In general, we meet logical difficulties when we try to define a mental fact by using a relation with a physical fact like, for instance, a cause-effect, or a stimulus-response. If we use the occurrence of the physical fact, that is one term of the relation, to define the occurrence of the mental fact, then the relation is not necessary, and we can use a direct mapping into

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48. This categorization is plainly acceptable because we required a bijection between the occurrence of a mental thing and the occurrence of the physical process that we used to define it.

49. The projections can be either onto the same subspace of phase space, or onto different subspaces.

50. We find: "Spoken words are the symbols of mental experiences and written words are the symbols of spoken words. Just as all men have not the same writing, so all men have not the same speech sounds, but the mental experiences, which these directly symbolize, are the same for all, as also are those things of which our experiences are images." *De Int*. I, 16a in *The Works of Aristotle*, W.D. Ross Ed.,Vol I, Oxford, 1963.

the physical fact. If we instead use a physical fact that we can consider as being in the stated relation with a certain other physical fact for defining a mental fact, then this physical fact still must be unique, otherwise we would ambiguously define the mental fact. However, here too, the relation is not necessary, because we can define the mental fact by mapping it directly into this last physical fact[51]. Clearly, we have an unsatisfactory definition when we define a mental thing by means of a relation to another mental thing, typically a mental category. We have no suitable individuation of the occurrence of a mental fact or activity when we use only mental categories applied to other mental categories: for instance, the mental category of cause applied to the mental category of something. We must substitute the something category with a physical fact whose occurrence shall be unambiguously identified, and shall be checked by means of repeatable experiments, in the system we are studying. However, when we decide to satisfy the constraints stated above, we are practically led to define mental things by means of an injective function into physical processes that occur in the system that we consider doing the mental activity.

This point is rather critical, and we prefer to add few examples. We start from the use of a cause-effect relation with a physical thing to define a mental thing. Let the physical thing that we use to define the mental thing be the effect in this cause-effect relation: that is, the mental thing is defined as being the cause of the occurrence of the physical thing. However, in the physical description it is related to another physical thing through a cause-effect relation: it is related to the physical thing that we consider as being the cause of its occurrence. Subtle problems of compatibility thus arise when we integrate the two viewpoints. When we are dealing with definition of mental things, if the occurrence of the physical thing that we use to define the mental thing is represented by the projection, onto a subspace of phase space, of a trajectory segment, then it can have different physical processes as possible cause. Therefore, our definition is not unique when we integrate the description of psychology with the physical description. On the other side, if the occurrence of the physical thing that we use to define the mental thing is represented by a segment of a trajectory in phase space, then our definition is again not unique. In fact, segments of different length, which precede the segment used in the definition, can equally be assumed as being the cause of the occurrence of this segment. Moreover, the mental thing so defined cannot occur again in the life of the same subject, because we assumed that the trajectories do not intersect in phase space. Therefore, in both cases we do not have a satisfactory definition.

We meet analogous problems when we try to use the stimulus-response relation. In a very simple experiment, where we ask the subjects to push a button when they see a flash of light, we have a correlation between a visual perception and a voluntary movement in a subject that was strongly conditioned to correlate the two facts; and the experiments on animals clearly show the necessity of this conditioning. At the level of a physical description we thus need a very sophisticated theory to disregard in the whole neural process the part that can be referred to the visual perception, the part that has to be ascribed to the conditioning, and the part that we can refer to the voluntary movement. At the level of a psychological description, we still need a sophisticated theory to predict how strong correlation the conditioning induces between the visual perception and the observed movement, because the interval time that we measure between the stimulus presentation and the voluntary movement, may depend on the strength of the conditioning too. The difficulties quickly increase when an injective function

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51. This point was not sufficiently stressed in R. Beltrame, "On brain and mind", *cit.*

fails to hold between the physical stimulus and the perceptual fact; and this situation is unfortunately the rule when the stimulus exceeds rather low levels of complexity.

We again meet analogous problems when we try to use purposive behavior to define mental things. When different activities bring to the same proposed result, we have the constraint to consider them as being equivalent means to achieve the proposed aim. This constraint is quite strong in our culture, although we have the possibility to maintain the differences among the activities by considering them as being a better or worst mean to attain the proposed aim, or by explaining why the subjects use different means to achieve the same aim; but this requires further mental activity. Hence, we cannot generally associate to an aim a unique activity as a mean to accomplish it. The existence of only one way to obtain a proposed result is matter of experiment, and we expect that this situation be very rare in biological systems, because the possibility to obtain a result through different trajectories of activity increases the life expectation of the biological system. The experience confirms this state of facts. For instance, we can take a book from our desktop by using different patterns of our muscles' activity; and we cannot assume that we have only one way to obtain the pattern of motoneurons' activity, which will lead to a certain pattern of muscles' activity, at least until we have experimental evidence of such a fact. From the above remarks we obtain a very sharp conclusion: we cannot use the subjects' aims to define mental activity because in general we do not succeed in setting an injective function of the mental things into the physical behavior.

Although the previous situations are not suitable to define mental things according to the requirements of this paper, they maintain a great interest for our discussion because subjects usually do not deal with the constitutive mental activity, but mainly concern themselves with relations among things that have the characteristics of the mental facts.

The dynamics of mental things

The injective function that we used to define mental things implies a one-to-one mapping between the occurrence of a mental thing and the occurrence of the physical thing that we used to define it. Therefore, we must choose the physical things among the physical changes and processes that we observe on the systems that we consider as doing the mental activity. At this point, we can investigate whether an isomorphism holds between the dynamics of mental things, and the dynamics of the physical facts that we used to define them.

We recall that we decided to define mental things by an injective function into physical processes. In a representation of the physical system dynamics as trajectories that do not intersect in phase space, mental things are thus defined through trajectory segment projections onto subspaces of phase space, and many trajectories can share the same projection. The main reason for defining mental things in such a way that the same subject will have the possibility to repeat them in different moment of his life (different segments of the same trajectory can share the same projection), and different subjects still have the possibility to repeat them (segments of different trajectories can share the same projection). We recall that the use of physical things in defining mental things is required by the repeatability constraint of scientific praxis.

As mentioned above, physics inherits from elementary classical mechanics the use of a mental scheme in which the cause of a body movement is external to the body itself. Therefore, we cannot assume a physical thing as causing its changes. In psychology, we instead use a mental scheme in which we think that what we consider as being a subject may be cause of its behavior, and in particular of its mental activity. Thus in psychology we think of the subjects as being the cause both of mental activities, and of physical changes in themselves and in their environment. Moreover, we think of the subjects as being the only producers of their mental activity: that is, we think that something else cannot directly produce the mental activity of a subject, though it can induce the subject to do a mental activity.

No methodological reason forces us to assume a different constitutive mental activity of the cause-effect relation in physics and in psychology. We thus assume the constitutive mental activity of the cause-effect relation to be the same, according to a general assumption about mental categories and categorization schemes. This assumption simplifies the definition of general notions, like cause and effect, although we have to set our definitions in such a way that they agree with the experimental results about the occurrence, in different contexts, of the physical processes that we used to define the categorization activity. As consequence of the previous assumptions, we must assume that different conditions hold in physics and in psychology to consider correct the choice of the things that we relate as cause and effect. No global isomorphism thus holds between the two dynamics, and we only can employ the designation of one order of things to indicate the other (because a one-to-one relationship holds between the occurrence of the two things), but we must be aware that the exchange only refers to names, and not to things.

When we view animals and humans as biological systems, we describe them and their behavior with the schemes of physics[52], and we automatically introduce the assumptions that are implicit in these schemes. We cannot thus maintain in this kind of description the scheme of psychology, because we would have to introduce the subject as cause of physical processes that occur in the thing itself, and this fact will lead us outside physics. Yet, we cannot base a psychological description of animal and human behavior on the schemes of physics. In these schemes, every change has its cause in something that is different from the thing that is changing. Then we lose the subject as it is thought of in psychology and its autonomy, whose consequences are today an essential character of the psychological description of human behavior. Nevertheless, the two schemes are both acceptable and useful. We can use them together, but we must be aware that we cannot freely transfer our deductions from one scheme to the other, and that we cannot claim that all the deductions of the two schemes hold together. Without this awareness, we may introduce contradictions, and we may easily show examples of this misleading possibility, for instance by referring to bacterial chemotaxis.

Motile bacteria will swim toward higher concentrations of certain chemical substances that we know to increase their life expectation (favorable chemicals), and they swim away from higher concentrations of chemical substances that we know to reduce their life expectation (noxious chemicals). Such a behavior is very subtly adapted and we know many details of it in *Escherichia coli (E. coli)*[53]. The bacterium swims by using flagella. Counterclockwise rotation of the flagella allows all the flagella to draw themselves together into a coherent bundle, and the bacterium swims uniformly in one direction. Clockwise rotation of the flagella causes them to fly apart, the bacterium tumbles chaotically, and its motion lacks statistically preferred direction. Without environmental changes, the counterclockwise direction of the rotation is reversed every few seconds for a brief interval of time, producing a characteristic pattern of

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52. Biochemistry, molecular biology, and electrophysiology are in fact grounded on physics.

53. See B. Alberts, D. Bray, J. Lewis, M. Raff, K. Roberts and J.D. Watson, *Molecular biology of the Cell*, 3rd Edition, Garland, New York, 1994, pp. 773-778, and the related bibliography.

movement in which a straight line is interrupted by abrupt, random changes of direction. Therefore, changes can be detected, which may occur in different places of the environment. When swimming at a constant velocity, the spatial gradient of chemical substances is detected as change in the chemical's concentration over time. If the concentration of noxious chemicals increases, then rotation reverses more frequently, thus inducing a more frequent change in the direction of the motion. If the concentration of noxious chemicals decreases, then rotation reverses less frequently, and the bacterium goes away from high concentrations of noxious chemicals. We observe an analogous procedure when the concentration relates to favorable chemicals. If the concentration increases, then rotation reverses less frequently, and the bacterium goes toward regions of higher concentrations of favorable chemicals. If the concentration decreases, then rotation reverses more frequently and the bacterium moves in different direction, but the frequency of reversing the rotation never goes to zero. So, even in favorable conditions, the possibility of better conditions is always explored.

For this bacterium we have rather detailed hypotheses to explain the observed behavior in terms of physical processes, starting from a small family of transmembrane proteins whose level of activation increases when they bound to a noxious chemical, and decreases when they bound to a favorable chemical. The activation induces a chain of chemical reactions. They involve the concentration of cytoplasmic proteins, and the multiproteins complex that acts as flagellar motor. The result is a clockwise rotation of the flagella and thus a tumble. The response time is about 200 milliseconds. We know many other details of the adaptation process that enables these bacteria to have a very good response. They can detect concentration changes over a range from less than 10^{-10} M to over 10^{-3} M for some favorable chemicals.

This very simple example shows that contradictions may arise when we decide to think of a behavior as being intelligent only when we consider the system as causing the occurrence of its behavior. In physics, we use a mental scheme in which the cause of a change is a different thing from the changing thing. If we decide to consider a behavior as not being intelligent when we think that its occurrence is provoked by a cause external to the system, then we should refuse to consider as being intelligent every behavior whose occurrence we explained in terms of physics. This conclusion will hold for more complex behavior as well, and in particular for human behavior; but the contradiction arises out exclusively from our pretension that two incompatible sets of conditions hold together: the conditions that we require to apply the cause-effect relation in physics, and the conditions that we require to apply it in psychology.

We might weaken the opposition between the two viewpoints, and even remove it, by changing one of them, for instance the point of view of psychology. It should however be necessary to reconsider a large part of our culture, which is based on the freedom of the acting subject, and, like ethics and criminal law, derive a statement of personal responsibility from this assumption[54]. This solution thus raises serious practical problems. Although the repeatability requirement in studying facts leads us to lose a private character of mental things as a consequence of their definition, we prefer to maintain two different theories of the behavior

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[54] The tendency to consider human behavior as being strongly dependent on external conditioning arose during quite recent criminal trials in Italy. The problem was recently discussed during a conference in Washington, *Neuroscience and the Human Spirit* sponsored by the Ethics and Public Policy Center, Washington DC, 24-25 Sept. 1998; see also "Does neuroscience threaten human values?", *Nature Neuroscience*, 1, 7 (1998), pp. 535-536. Another related viewpoint is the distinction between the faculty of understanding and the faculty of will in discussing whether the persons are in full possession of their faculties. Finally, this point is also related to the discussion given below on the paradigms used in doing mental activity.

for the systems that we consider as being able to do mental activity: a physical, and a psychological theory. Our program of integrating these two theories will have as necessary link only the injective function between physical and mental things that we used to define mental things.

Memory phenomena

Memory is another field that shows deep differences between the approach of physics and that of psychology. When we take the approach of physics, we must be aware that we decided to concern ourselves with an isolated physical system[55]. Furthermore, we decided to represent the interactions that we use as elemental in the dynamic of the isolated system by conservative forces that do not depend explicitly on time. In this context, we usually speak of memory phenomena[56] when in the theory that explains and predicts the behavior of the system, the values of the variables that we have defined depend on the values that these and other variables assume both at the same instant of time, and on the past. Memory then becomes an aspect of the dynamics of the system, and we systematically will refer to this characterization when we are dealing with physical description. When we take the approach of psychology, we also have to deal with conscious memories

In physical description, we can relate memory phenomena to the mutual movements of the parts of the system that are involved in elemental interactions, and to the interaction delay when elemental interactions relate distant parts of the system.

The first scheme considers that the parts of the system change their mutual positions, that the strength of the interactions still changes, and so the system shows a different behavior. We have a so direct link with the dynamics of the global system in this scheme, that it is difficult to separate the description of memory phenomena from the description of the dynamics of the system. We can link this viewpoint with the more usual one, in which memory phenomena are explained through change in the material that composes the physical system, by recalling that the spatial configuration of the interacting parts defines the material and its characteristics. We can introduce a locality principle[57]: that is, the changes in each part of the system depend on what happened in the past time only to that part and to its immediate neighborhood. It is acceptable because it has an experimental basis, and because we can deduce it by a definition of the architecture in a more analytic theory[58]. Furthermore, the architectural changes must be interpreted in a broad sense. For instance, we can invoke different concentrations of certain molecules in a part of the system for explaining why the same process causes different processes; or we can have a protein *A* that is a gene regulatory protein in activating its own transcription. If an action turns on the expression of the protein *A* in a cell, then all the descendants of that cell will produce the protein *A*[59].

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55. Recall that we require the system to be isolated in order to predict the energy exchanges. By isolated system we mean the biological system plus a part of its environment such that their sum could be considered as being an isolated system.

56. A certain number of problems discussed in this section were discussed in R. Beltrame, "Memory and mental activity", *Methodologia*, 12/13 (1993), pp. 173-180.

57. See C. Truesdell, *A first course in Rational Continuum Mechanics*, Vol. I *General concepts*, New York,1977; and M. Silhavy, *The Mechanics and Thermodynamics of Continuous Media*, Springer, 1997.

58. A concise discussion of this point can be found in C. Truesdell and W. Noll, *The non-linear field theories of mechanics*, *cit.*, Sect. 3.

59. See B. Alberts, D. Bray, J. Lewis, M. Raff, K. Roberts and J.D. Watson, *Molecular biology of the Cell*, *cit.*, p. 444, and the related bibliography. Nevertheless, these last types of phenomena may also be formalized as phase transitions.

We think of such a type of modifications as being stable: that is, we assume that their effects on the behavior of the system will be maintained until further modifications occur in the material; and this is a further consequence to describe them through elemental interactions. The technique often forces this aspect, and it offers several examples of objects in which this way of considering a physical system with memory is particularly evident: for instance, the magnetic disks commonly employed in computers.

Now, let us think of a physical system as being composed of interacting parts, and let us refer to a scheme in which we consider the change in a physical quantity at a certain point of the system as being the cause of the changes of the same or of another physical quantity at a different point in the system. If we decide that it is significant the delay between the occurrences of the two processes that we relate as cause and effect[60], then the value of a physical quantity at a certain point and time will depend on the values that the same or other physical quantities assumed at different points and at past instants of time. This scheme, that we will call interaction delay, offers a further way of describing memory phenomena in physical systems, and it is noteworthy that, in this conditions, the system shows phenomena of memory without us having to assume changes in the material and the architecture of the system[61].

If the interaction between the parts of the system is active for a long time, then the past values of the variables that affect the actual value of the observables, may still depend on the values that certain variables assumed in other points at earlier instants of time, and so on. We must describe memory phenomena in a way that does not violate, even implicitly, the repeatability of the experiments, and this implies that elemental interactions do not depend explicitly on time[62]. However, we cannot consider the delay in interaction as being an explicit dependence of the interaction on time. Delay is a characteristic of the physical phenomenon that we consider as interaction, and its properties follow from specific experiments[63].

Technically, the state of the system now may depend on the history of the system, because we may have a back propagation chain of dependencies on the past activity of the system. We easily assure the repeatability of the experiments when we succeed in defining state variables, because the knowledge of the value of these variables is equivalent to the knowledge of the system's history for predicting the future behavior of the system. Probably, the best example of this technique is in elementary Newtonian mechanics. In this theory we have a vector whose

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60. When the effect in the interaction follows the cause with a certain delay, it is usual to speak of delayed action, or of delayed interaction, both when the cause and the effect occur at the same point, and when they occur at different points. When the delay is considered significant, and when cause and effect occur at different points, it is often satisfactory to express this delay as a linear function of the distance between the two points where the changes of the physical quantities occur; and, in this situation, the term 'propagation speed' designates the constant rate in the linear function. Nevertheless, the reasons for introducing this concept in a theory, with the related problems about a thing that would travel from one point to another, really relate to the decision to write equations of balance for certain physical quantities, which must hold at every instant of time both for the system, and for its parts. A very good discussion on this point can be found in R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman lectures on Physics*, *cit.*, Vol. II, pp. 27.1 ff.

61. We recall that the occurrence of memory phenomena of this type is very frequent in natural systems. Systems without memory are nevertheless of theoretical interest because of their simple mathematical treatment, and because the actual production of the artifact relates to systems with a behavior strictly stereotyped, repetitive: that is, a behavior that we want to be independent of the system's history.

62. More generally it must be invariant for translations of the completely ordered parameter that we use to describe a process. The interval [0,1] of the set R of real numbers is usually assumed as a prototype of the formalization of this parameter.

63. When we assume this viewpoint, it is not strange that the delay in electromagnetic interaction only depends, in certain conditions (a motion of the interacting bodies that does not change their distance and whose velocity is constant), on the distance between the two interacting bodies, and it does not even depend on the direction of the velocity, as the Michelson-Morely experiment proved.

value corresponds to a linear functional of the history of the forces that acted, over a certain interval of time, on the mechanical body that we assume as being atomic. This vector is the momentum[64]. Unfortunately, there is no general method to define state variables, and it is usually difficult to define suitable state variables in a complex system. Nevertheless, when we assume a methodological viewpoint, this strategy seems to me the only safe strategy. In the reference physical theory, which was essentially introduced to have a clear methodological discussion, we will assume that we succeed in defining suitable state variables, because we assumed that elemental interactions have potential.

We can satisfy the repeatability constraint also when a knowledge of the system's history over a limited interval of time is sufficient to formulate predictions about the future behavior of the system: that is, when we can limit the back propagation chain of the dependencies to a reasonably short interval of time. Furthermore, when for a sufficient interval of time two parts of the system do not interact, the effects of the previous type of memory also cease on these parts. Let interaction involve only few parts of the system, and let these parts change their activity. The interaction delay can exhaust its effects. We thus can predict a decay of this type of memory when the activities alternate, and when they involve interactions among disjoint parts of the system, or, at least, when they have as target disjoint parts of the system. We thus expect this effect when we alternate different activities, and we know that such an alternation usually reduces fatigue. Both men and other mammals alternate periods of conscious activity with periods of sleep.

In biological systems, we have cells that die and new ones replace them at rather regular intervals of time. The constituents of the cell are regularly replaced with new ones with very few exceptions. At the level of a single cell, we have, for instance, continuous phenomena of endocytosis and exocytosis, and, more generally, soluble, or secretory proteins, and other substances are thrown in the intracellular space, and are imported from it. Newly synthesized plasma membrane lipids, transmembrane proteins, and cytoplasmic proteins replace the old ones. Cytoskeleton has a dynamics, and so on. Indeed many of these processes can be also receptor-mediated, and so actions of the cell's environment can modulate them[65]. We may think that these substitutions of old biological material with new material contribute to canceling the link with the past activity that is induced by the interaction delay.

Moreover, let our system occupy a region of space such that we can neglect the delay of the interaction. We can simplify the study by substituting the knowledge of the external actions with the knowledge of the values that significant physical quantities assume on a closed surface that envelops the system. This strategy is frequent in laboratory experiments. However, we must have a satisfactory theory that predicts the values of these physical quantities on the closed surface that envelops the system[66], and that predicts the energy flow across this surface. Otherwise, we do not succeed in integrating the experimental result into a more general theory, or in transferring the results from *in vitro* experiments to *in vivo* systems. Finally, in integrating the physical and the psychological descriptions of memory phe-

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64. Velocity becomes a state variable when the mechanical body is also assumed as having a constant mass.

65. About endocytosis and exocytosis traffic at the level of single cell a good starting point is in B. Alberts, D. Bray, J. Lewis, M. Raff, K. Roberts and J.D. Watson, *Molecular biology of the Cell*, *cit*., Chap. 13. However, there is a rapidly increasing bibliography on the replacement of the cell's constituents, and its dynamic.

66. Another way of describing this difficulty is that we must know which physical processes produce the observed values of the physical quantities on the closed surface that envelops the laboratory system. Otherwise the relation between the dependent and the independent variable only holds for a pointwise value of the physical quantities on the closed surface that envelops the laboratory system.

nomena we must be aware that we can easily define a mental thing by a mapping into a physical process; but we have difficulties to map it into a system condition, like a pattern of molecules' concentration. We proposed to avoid this difficulty by introducing the process that brings the system to that state; but we usually need a very detailed knowledge of the system dynamics for using this strategy.

Psychological descriptions use different ways of considering memory phenomena, and here, we will discuss conscious memories. We instead will discuss learning later, in the framework of the constraints on mental activity.

When we speak of a mental fact as being a conscious memory, it was proposed that we consider it as being repetition of another mental fact and as having occurred in the past[67]. Following this hypothesis a mental fact becomes a conscious memory through a mental categorization that follows the scheme described above. Two sets of conditions thus constrain the occurrence of a conscious memory. A first set relates to the possibility of executing the constitutive activity of the mental fact that should be the content of the conscious memory. The second set relates to the categorization of this mental fact as being a repetition of a mental fact that occurred in the past to the subject of the conscious memory. We thus expect that conscious memories selectively shall be lost of the facts that subjects cannot produce for any reason, although the facts occurred many times in their life. Achromatopsias are known, for instance, which follow from brain lesions, in which an adult man loses the ability both to perceive and to remember colors, even if he had perceived and remembered colors several times before incurring the disease.

From thinking of one thing as being a repetition of another, we also expect that subject thinks of the two things as being equal. Furthermore, the proposed categorization scheme implies that in the comparison, we use as paradigm what subjects think to have occurred in the past. We become aware of this fact when a disagreement with this paradigm (for instance by means of factual or document checking, testimonies, etc.) leads subjects to explain by suitable causes the failure of the consequences that they expected by having applied the mental categories. When no check occurs, the subsequent behavior continues as if the expected consequences held[68].

This quite general behavior has particular relevance in our case. The stimulus is weakened to check whether the conditions hold to apply the categorization scheme proposed for the conscious memories, and this effect will become progressively stronger when the situation repeats. Motivations, of which the person might not be completely aware, can strengthen the tendency to avoid any check. Moreover, a subsequent memory can base itself on a previous one, rather than on the original situation, because subjects assume the actual mental activity to be a repetition of one that occurred in a previous memory. They thus apply a transitive property. Therefore, we can have facts that the subjects consider as being good memories, but they may not have occurred, or they may reveal significant differences from the facts that subjects consider as memories when someone checks them. Since subjects consider these facts

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[67] This characterization was proposed in S. Ceccato, *La fabbrica del bello (The farm of beauty)*, Rizzoli, Milano, 1987, pp. 234-36 (in Italian). It is also interesting the Aristotle's discussion on this point in *De Memoria*, *Parva Naturalia*, 450a.25 ff. We emphasize that we speak of conscious memory about mental facts, and we do not speak of conscious memory about mental activity. Therefore, I think that we must preliminarily consider something as being a mental fact through a suitable mental categorization.

[68] We avoid talking about consequences that are assumed to be true or verified, because a check is implied, which was excluded by hypothesis.

as really occurred in their past life, we may have relevant consequences on their behavior, and these consequences may reach the mental disease.

It is interesting to apply the previous way of considering conscious memory, to a crucial point in Freud's development of psychoanalysis. Freud reports that one of his patients remembered, under analysis, seduction situations (that is, passive sexual experiences) that he claimed to have suffered during his childhood; but these memories turned out to be untrue when a later check was made on the patient's history[69]. We know that gestures of affection frequently may assume sexual connotation after the sexual differentiation is completed in adolescence, and this fact can arise more frequently when these gestures involve the tactile sensory system. Let a person starts from the memory of an affection gesture that involve the tactile sensory system, and that relate to his childhood. He now can feel this gesture as having a sexual connotation. If he considers this situation as being a repetition of what occurred during his childhood, the scheme proposed above shows that this situation becomes a memory of what the person felt during his childhood. Through an analogous mechanism, the subject may attribute a sexual feeling to the person who made the affection gesture. Persons must use thought to conclude that their actual feeling differs from their feeling during childhood, and they must agree with a paradigm that is transmitted by culture. The paradigm content is precisely that the biological process of sexual differentiation during the adolescence introduces a difference in our feelings, so that we normally must refer sexual attraction or repulsion only to persons that have reached this level of sexual differentiation[70].

We may try to involve mental categorization for explaining other memory functions that we meet in psychology. A subject, for instance, can consider that a thing is the same that occurred in his history, and this categorization might be a good candidate for the type of recalling that our languages indicate by using pronouns or definite article.

We also have to explain why in a certain moment a person considers a mental fact to be the repetition of a past fact, and why persons sometimes report facts that are the same as the ones that occurred in their life, and sometimes facts that do not[71]. These problems are however part of the more general problem of describing and predicting the flow of the mental activity.

The flow of mental activity

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[69] We will quote Freud's first communication in his letter to Fliess of September 21, 1897: «Then the surprise that in all cases, the father, not excluding my own, had to be accused of being perverse - the realization of the unexpected frequency of hysteria, with precisely the same conditions prevailing in each, whereas surely such widespread perversions against children are not very probable. The incidence of perversion would have to be immeasurably more frequent than the resulting hysteria because the illness, after all, occurs only where there is a contributory factor that weakens the defense. Then, third, the certain insight that there are no indications of reality in the unconscious, so that one cannot distinguish between truth and fiction that has been cathected with affect.», and later he notes: «It seems once again arguable that only later experiences give the impetus to fantasies, which hark back to childhood, and with this factor of a hereditary disposition regains a sphere of influence from which I had made it my task to dislodge it - in the interest of illuminating neurosis.» S. Freud, *The complete letters of Sigmund Freud to W. Fliess 1887-1904*, transl. J.M. Masson, Harward University Press, Cambridge, 1985, pp. 264-5.

[70] This picture still agrees with Freud's analysis of pathological behavior. The critical point is the equilibrium between thought and sensations. Moreover, when the cultural paradigm is not accepted, the subjects may suffer the consequences to think of the sensations that they attribute to the partners of the imagined sexual act. Depending on the distribution of pleasure and repulsion between the partners, we can find the wide range of reactions that Freud and other psychoanalysts clearly described and studied.

[71] Clearly equality and differences result here from a comparison between what a subject reports as a memory, and the contents of a physical record of the fact which the subject is talking about.

The equations that describe the evolution of a physical system completely describe its dynamics and the flow of its activity. Different configurations of the values of the observables that characterize a state of a system completely characterize different evolutions, because we decided to refer to a deterministic description. Furthermore, since we decided to describe the flow of a physical process as being continuous, we can geometrically think of a physical process as a continuous line in a space having a suitable number of dimensions: the phase space. It is important for our discussion on the flow of mental activity to remember that a physical system can remain for a long time at the interior of a certain range of physical conditions without violating the requirements of a deterministic description of its dynamics. In fact, a trajectory can remain for a long time at the interior of a small volume of phase space without intersecting it or other trajectories.

The approach of psychology gives rise to a description of the activity flow, which is conceptually more complicated than the physical description. We decided to define mental things through an injective function into physical processes that are characterized by a lower number of parameters than those which are necessary to give a deterministic description of the system dynamics. The occurrence of mental things now requires as counterpart the occurrence of only a part of the physical processes that we introduced to predict deterministically the occurrence of a physical process. Many trajectory segments can share the same projection onto a subspace of phase space. Therefore, the occurrence of what we defined by a projection can be obtained by doing the physical activity that we described by the trajectory segments that share the projection[72], and the same mental thing can be realized in different contexts of activity and by a different biological architecture.

This situation holds for the dynamics of all the things whose definition involves only a part of the processes that are necessary in the reference physical theory, irrespective of the fact that these things are mental things. We frequently meet situations of this type in psychology, because human behavior is typically defined. For instance, we define in this way the muscles' activity that is responsible for the utterance of the words and the sentences of our languages.

We will discuss the consequences of the previous decisions by referring to mental things, although a large part of the results will hold for all things whose definition involves only a part of the processes that are necessary to have a deterministic dynamics. Furthermore we will refer to the properties of a dynamics that can be represented in phase space by trajectories that do not intersect. We recall that in this picture, mental things mirror the properties of projections, onto subspaces of phase space, of one or more segments of the trajectory that describe the evolution of the system in phase space.

In discussing the flow of mental activity, it is important to remark that projections can have many configurations. For instance, they can be projections of the same segment onto different subspaces of phase space, or they can be projections of successive sub-segments onto the same or different subspaces, and we must define the timing pattern of their occurrence as well. We thus have many possibilities when we define a thing in this way. The simplest one consists in defining the occurrence of a mental thing by the occurrence of a process whose geometrical representation is a segment of line into a subspace of phase space, we mainly will deal with this simplest scheme, because the methodological aspects of the problem are equally

[72]. This fact shows a further source of the effects that we usually ascribe to the plasticity of the nervous system, and this source does not require the local changes in the biological system architecture, that we mentioned in discussing memory phenomena. In particular, we are not required to introduce new learning activity.

clear.

If we prefer to avoid this geometrical representation, we can simply reword the previous picture in the following way. The trajectories in phase space become the description of how the physical system evolves from certain initial conditions, and the equations that describe this evolution fully describe the dynamics of the system. The trajectory segments in phase space become the physical processes that we introduced in our theory to explain deterministically the behavior of our systems. Their projections become the sub-processes of the physical processes that we used for defining the items of the psychological theory in such a way that they can occur again in the life of the same subject, and that they can occur to different subjects.

We can have trajectory segments that are orthogonal to certain subspaces of phase space, they project themselves onto these subspaces as points, and we do not consider these projections as representing a process. When the system traverses these trajectory segments, we do not observe occurrences of processes that we defined as lines into the orthogonal subspaces. We thus can explain through the shape of the trajectories in phase space, why a system acquires or loses a behavior that we defined by using a projection of trajectory segments, and we can predict how long these changes last. We emphasize that effects of this type do not necessarily require changes in the architecture of the biological system, because the shape of a trajectory segment typically depends on the interactions among the parts of the enlarged system. We recall that this enlarged system encompasses the biological system and the part of its environment that allows us to consider this extended system as being an isolated physical system.

Determinism generally does not hold between the occurrences of facts whose definition uses only a part of the physical processes that we defined for developing a deterministic physical description[73]. Limited forms of determinism are however possible, and we can give the following geometrical representation to this local determinism. Let A and B be the processes that define the facts that we wish to consider respectively as being the cause and the related effect, and that are represented by line segments in certain subspaces of phase space. We have a local determinism when all the trajectory segments whose projection is B follow a segment of the same trajectory whose projection is A, and all the trajectory segments whose projection is A precede a segment of the same trajectory whose projection is B. We also must require that a trajectory segment whose projection is different of B does not follow a segment of the same trajectory whose projection is A, and a trajectory segment whose projection is different of A does not precede a segment of the same trajectory whose projection is B. If the definition of A or B involves more projections, these conditions must hold for all projections, and their timing pattern. Finally, when we claim for a chain of cause-effect relations these conditions must hold for all the trajectories that share as projection one of the elements of the chain.

This picture shows that local determinism critically depends on the shape of the trajectory segments. It mainly depends on the interactions that are currently active in the enlarged system, and on the history of these interactions, because we usually have memory phenomena. It also depends on the state of the enlarged system, although this dependence may be weak when a trajectory remains for a long time in the same volume of phase space, because many trajectory segments can satisfy the same subset of physical conditions. Finally, we expect that the more the situation is complex the less is the probability to have local determinism, since trajectory segments that satisfy local-determinism conditions have a high probability to be fol-

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[73]. We recall that determinism requires the use of one-to-one cause-effect relations

lowed by segments that do not satisfy these conditions.

We can introduce a general strategy if we accept to describe only a correlation between the occurrences of the things so defined that a global determinism does not hold. However, we must be aware that we usually renounce to individuate what determines the existence of the correlation, and our optimal result is to show the conditions under which a correlation holds. Moreover, when we try passing to a deterministic approach, we must be aware that we need further information, because both the correlated facts may be effects of a third fact[74]. This point is very critical when we interpret a scientific result or when we forecast its practical applications, and the geometrical picture that we are using offers an intuitive explanation of the situations that we can meet. Let the correlated facts mimic the properties of projections, onto suitable subspace of phase space, of trajectory segments[75]. Since many trajectory segments can share the same projections, we cannot surely predict which processes follow the correlated facts, and we can meet unexpected results.

For instance, we inherit from the history of biology an evolution scheme in which a new behavior grows together with related changes in biological architecture. In these cases, it is usual to correlate the two orders of facts. Since the state that corresponds to the architectural changes has the characteristics of a projection, many trajectory segments can share it. Therefore, different physical processes can reach it, and we know that different techniques of biochemistry or of molecular biology can induce the same architectural changes. Moreover, correlation depends on the characteristics of the trajectory segment. Therefore, the correlation may hold when the state belongs to certain trajectory segments, and it cannot hold when the state belongs to other trajectory segments. A new behavior can thus be obtained when an activity of the system induces certain architectural changes, and it may not be obtained when we induce the same architectural changes through techniques of molecular biology[76].

When we use a probabilistic approach, we must define the probability that each projection occurs starting from the projections through which we defined a mental or psychological fact. Mathematically, this probability is related to the ratio between the number of trajectory segments that share a projection[77], and the number of trajectory segments that share a greater number of properties. The choice of this last set of properties defines the possible segments, and it usually depends on the problem we are dealing with. For instance, we can choose the number of trajectory segments that share the beginning of the projection segment, but we can impose more strict conditions. We can try to use the same class of possible processes to define all the probabilities, and we thus will obtain a more general theory. However, I think that a bot-

[74.] In physics, for instance, we use a correlation between two facts for defining an interaction between two things, but the interaction that we defined in this way must be used as atomic in the theory.

[75.] If the correlation would be between trajectory segments, we should have a deterministic situation.

[76.] Some interesting results are reported in V. Porciatti, T. Pizzorusso, and L. Maffei, "Vision in mice with neuronal redundancy due to inhibition of developmental cell death", *Vis Neurosci* 1999 Jul-Aug;16(4):721-6. They experimented with transgenic mice overexpressing *bcl-2*, which, due to inhibition of naturally occurring cell death, have much larger brain and optic nerves as compared to wild type mice. By recording Local Visually Evoked Potentials (VEPs) from the primary visual cortex in response to patterned stimuli, they found that the representation of the visual meridian was displaced by about 15% in the *bcl-2* mice, but visual acuity, contrast threshold, and response latency were normal, indicating that compensatory mechanisms can ensure normal basic properties of vision in spite of marked neuronal redundancy. See also Porciatti V, Pizzorusso T, Maffei L , "The visual physiology of the wild type mouse determined with pattern VEPs", *Vision Res* 1999 Sep;39(18):3071-81. Other behavioral experiments of the same laboratory show that *bcl-2* mice have normal behavioral performance in a T-maze apparatus (L. Gianfranceschi, A. Fiorentini, L. Maffei, and V. Porciatti, "Behavioural visual acuity of wild-type and *bcl-2* transgenic mouse", *Society for Neuroscience Abstracts*, 1996,1060)

[77.] For simplicity sake, we can think of a single segment of line, but we can have more complex types of projection, as we discussed above.

tom-up approach should be preferable in this type of problems. Furthermore, suitable conditions must hold to ensure that this ratio has a finite real number as limit, and these conditions constrains the dynamics. In particular, we may require ergodicity to simplify the link between the theory and the experimental results, though ergodicity is rather difficult to prove as property of a physical system. We will not concern ourselves with other details of a probabilistic approach to the dynamics of our systems, because the mathematical aspects often would require a sophisticated use of the tools.

The occurrence of a long chain of facts is less probable than the occurrence of a shorter one when the facts were defined through projections of trajectory segments, and we expect less coherence in the system's behavior when it involves long chain of facts. This remark agrees with the experimental results: we observe coherence on the subjects' behavior in the short period, and lack of coherence in the long period. Since an identical repetition of a long chain of mental facts is practically impossible, subjects frequently shorten the chain of facts that constrains their behavior, still because they have constraints on time. We thus observe that the behavior is motivated by a less number of elements than the number that we introduce in the general theory. The motivating elements may change from one occurrence of the decision and another, because subjects use the mental facts whose occurrence is more probable in that moment.

The previous conclusion still applies to motivation and drivers because we defined them by using only a part of the processes that were necessary to predict deterministically the flow of the physical activity. Therefore, they can occur again in the life of the same subject, and they can occur identically in different subjects, but such a definition excludes a deterministic prediction of their occurrence. The properties of motivation and drivers mirror the properties of projections, onto subspaces of phase space, of trajectory segments. Many trajectories of phase space thus can share a projection, and we need further conditions to predict deterministically the occurrence of a motivation. However, we cannot use other motivations or drivers, because we will have a *regressum ad infinitum*, and several motivations and drivers usually can lead to the occurrence of the same mental thing. Finally, further physical processes occur besides those involved in the definitions. These further physical processes may differ from one occurrence to another and they may have no counterpart in a psychological description, but they may be essential to explain the occurrence of other, subsequent effects, and among these effects, we can have the occurrence of mental facts. The consequences are rather subtle, and they are not well studied. For instance, we can meet them in clinics, as psychosomatic effects or diseases.

We can use the geometrical picture to clarify the dynamics of other classes of facts. We consider the projection of a trajectory segment onto a subspace S_1 of phase space; the next trajectory segment will be orthogonal to S_1, and we consider its projection onto a subspace S_2 of phase space. Finally, the next trajectory segment will be orthogonal to S_2, and we consider its projection onto a subspace S_3 of phase space. This situation can depict the scheme of our thinking of two things in a certain mutual relation: for instance, a cause-effect, or a part-all relation. We may use the projection onto S_1 to define one of the things that are in mutual relation, and to define a part of the constitutive activity of the mental category that characterizes the relation. We may use the projection onto S_3 in an analogous way with respect to the second thing that is related to the first one. We may use the projection onto S_2 to define the remaining part of the constitutive activity of the mental category that enters to constitute the relation. Clearly, we might have a more complex scheme of subspaces and of timing patterns, but we prefer to mention only another related scheme, and to point out the essential role that the definition of mental facts acquires in these schemes.

Subjects can speak of a relation between things that occurred in different moments of their life, and the related things may be separated by a temporal gap. We can apply the previous scheme to the moment in which subjects speak of the two things in mutual relation, but more complex schemes are possible because the related things may be conscious memories. We will come back on these points in the next section.

When we consider a single trajectory, we can use the shape of the trajectory in the neighborhood of different points to explain a different correlation between the projections. This remark allows us to describe learning as the effect of an activity – the training – whose occurrence brings the system into a region of phase space where certain projections of trajectory segments of trajectory have a higher correlation. These projections are the physical processes that we used to define the things that learning involves. We emphasize that learning belongs to psychological descriptions in which it is often introduced as a faculty. In physical description, we only need changes in the physical architecture of the system. In explaining learning, we must start from the physical description of the system dynamics. We must describe the changes that training induces on the physical architecture and the effects that these architectural changes induce on the successive physical activity of the system. Finally, we have to choose our definitions of the things that learning involves so that we obtain the observed correlation between their occurrence before and after training. Training is still defined through physical processes that map into projections of trajectory segments. Since many trajectory segments can share a projection, we cannot predict deterministically the effects of training and its effectiveness as well. The effects depend on the current trajectory segment, and we meet a well-known character of learning: its strong dependence on individuals, and on their conditions. Teaching acquires the same characteristics as well.

Another consequence of our way of defining mental activity is the double attitude that we can assume versus its occurrence. We can focus on the occurrence of the physical process that we used to define the thing, either as the occurrence of the defined thing, or as embedded in the full process that realizes its particular occurrence. In the first case we stress the repeatability of the thing. In the second case we stress the determinism and the singular aspects of the particular occurrence, but we must go back to the physical description. We can alternate the two viewpoints in successive moments; but we cannot freely mix the deductions that we obtained from them, and this fact is immediately evident in literary criticism.

We only mention the case of linguistic behavior. We can focus ourselves on the relation between designating and designated thing as it is defined in a particular language, or we can focus ourselves on what we think that our interlocutor wished to communicate. In the second case, we consider the previous relation between designating and designated thing as being the way of communicating, and we usually must infer which previous mental activity led a person to speak or write a certain sentence. We usually infer this knowledge from the previous and the next physical activity of our interlocutor.

Many conclusions of the previous discussion apply to physical things so defined that their properties mirror the properties of projections of trajectory segments. Experiments only can decide in which extension and details the consequences that we discussed above apply to the different cases, because we are dealing with physical processes. However, we meet here a critical point, since the previous conclusions still can apply to many biological processes in a cell.

Constraining mental activity

As we have seen, when we take the viewpoint of physics, the equations that describe the dynamics also describe the constraints on the flow of the system activity. We can take an analogous approach when we look at the flow of mental activity, although we cannot predict deterministically this flow for the methodological reasons that we discussed in the previous section. However, the flow of mental activity can be discontinuous. We can justify this conclusion by remembering that mental activity has properties that mirror those of projections, onto subspaces of phase space, of trajectory segments.

Projections, as we have seen, can assume many configurations. Let a flow of mental activity be defined through a physical process that is represented by a segment *MN* of line in a subspace *S* of phase space. This flow occurs when a trajectory segment occurs in phase space, which has *MN* as projection onto the subspace *S*. We can have two cases. No part of the trajectory segment is orthogonal to the subspace *S*. Some parts of the trajectory segment are orthogonal to the subspace *S*. In the first case the inverse function of *MN* into time is continuous, like the analogous inverse function of every trajectory segment in phase space. In the second case this function is not continuous, and we can describe the flow of mental activity as being composed by at least two spans of mental activity that are not temporally contiguous. Moreover, the previous case is only a simple example of a large class of more complex situations.

If we plan to describe the constraints of mental activity, we practically must go back to physical description, and we must adapt the constraints on trajectories to the projections that we used to define mental activity. We also can see culture as source of constraints, and these constraints have a conceptually different character than those of physical activity because they refer to spans of activity: that is, when we deal with mental activity, they refer to mental facts. In discussing these constraints, we will distinguish between constraints on a span of activity that flows without interruptions, and constraints on an activity that flows with a certain number of interruptions.

This dichotomy is essentially a scheme of discussion, because the occurrence of a span of activity frequently mixes the two types of flow. Walking is a good example of this situation, and it is often presented as an example of hierarchical scheme of motor control in neurophysiology[78]. In this framework, walking is described as an activity in which the details of the muscles control do not require a conscious intervention of the subject, who, for instance, can freely think during walking; and we can describe this muscles control as iteration of a procedural memory item. However, when an obstacle, or another cause, unbalances the body more than a certain amount, we usually have a shift to an activity that corrects the posture, and that often has the character of a conscious activity.

In discussing these constraints, we must remember that a suitable psychological theory of human behavior has to satisfy the common assumption that considers this behavior as being anomalous when we observe a flow of small and disconnected pieces of mental activity: that is, when the behavior has a severe lack of stability and of coherence. We also consider an excessively stereotypical behavior as being equally anomalous, and we impute it to a poor mental activity of the subjects, or to an excessive polarization of their mental activity. Therefore, a satisfactory description of systems to which we attribute a sophisticated intelligent

78. See, for instance the part devoted to the control of movement in E.R. Kandel, J.H. Schwartz, and T.M. Jessel, *Principles of Neural Science*, 3rd edition, Elsevier, 1991, pp. 533 ff.

behavior, like human beings, must be equally far from these two extremes. If we assume the course of mental activity as being constrained, we may avoid psychological theories in which the behavior is too fragmented and disconnected. If the scheme of constraints is sufficiently rich and flexible, we still can avoid theories in which the behavior would be too stereotyped.

Training is the main source of constraints, and we previously showed a possible interpretation of training in terms of the physical description. When we think of procedural memory items whose activity is defined as flowing without interruptions[79], the classical Pavlovian conditioning is a way of realizing this training. It is usually described as follows. An indifferent cue, for instance a flashing light, when properly paired with an unconditioned stimulus (US), for instance a shock, can be trained to elicit some of the consequences of the US as a conditioned response (CR), for instance various indices of fear, and so the original indifferent cue becomes a conditioned stimulus (CS)[80]. Other training methods involve mental activity, and they are usually described as the repetition of a voluntary activity until the subject becomes able to do this activity without driving it consciously. We will point out some differences between these two types of training.

In Pavlovian conditioning we have a new stimulus that elicits the same response of the unconditioned stimulus. So, we only have a new correlation among facts. Mathematically, the mapping of the CRs into the CSs is not a single value mapping, and this means that we cannot infer unambiguously the stimulus from the response. This fact prevents us from using, as a general strategy, stimulus-response relations of this kind to define a mental thing. We cannot, for instance, define a mental thing as being the stimulus of a physical thing that we consider as being a response. We instead think that voluntary actions (for instance, voluntary movements) usually have a goal, and so we usually explain their occurrence by a final cause. However, a final cause is not useful to test predictions about the occurrence of something, and an efficient cause is required to plan a scientific experiment. In a psychological description, the efficient cause of a voluntary action is usually the subject who is doing it, or a faculty, like willpower. In a physical description this solution cannot be accepted, because, by definition, the change must be induced by a physical thing different from the thing that is changing. So, we meet again a break of a possible isomorphism between the physical and the psychological approach to the dynamics of our systems.

Moreover, learning seems complex still when training follows the scheme of Pavlovian conditioning. According to the general picture that we are using on this paper, we must describe training as repeating the projection of a trajectory segment, and this brings the system in a

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79. We can think of such a kind of procedural memory items either as a single process of suitable complexity, or as two or more concurrent processes. As mentioned above, we can freely choose one or the other scheme, however in biological systems we have some problems. The scheme of concurrent processes is really useful when no interaction affects the parallel processes between two subsequent synchronization points. When, like in mammals, we have endocrine and immunological systems that are highly pervasive, we must carefully check that a decomposition in concurrent processes satisfies the requirement indicated above.

80. The conditioned stimulus theory has to explain the situation described in the text, to which we refer here as Experiment 1, and these further experimental results. Experiment 2 points out that if two equally salient cues, for instance a flashing light (CS1) and a tone (CS2), appear simultaneously during conditioning trials before the shock (US) occurs, then each of the cues can separately elicit a fearful reaction (CR) on recall trials. Experiment 3 is constructed by doing Experiment 1 before Experiment 2. When the tone (CS2) is presented on recall trials, it does not elicit a fear reaction (CR). Experiment 4 is like Experiment 3, but the US is varied in the compound trials. For example, the US1 which follows the light (CS1) is a prescribed shock level, and the US2 which follows the compound light and tone (CS1+CS2) is a sufficiently different shock level. If US1<US2, then the tone elicits a fear reaction, whereas if US1>US2, the tone elicits a relief reaction. See, for instance, S. Grossberg, "How Does a Brain Build a Cognitive Code", *Psych. Rev.*, 87 (1980), pp. 1-51, with the related bibliography.

volume of phase space where the correlation is high between the conditioned stimulus (CS) and the conditioned response (CR). However, we must explain why learning strengthens the correlation between the conditioned stimulus and the conditioned response, although in training procedure we have the occurrence of an unconditioned stimulus (US) between the CS and the CR. The changes in the state of the physical system are produced by iterated occurrences of processes that have as projection the physical counterpart of CS+US. We thus need a very articulated knowledge of the system's dynamics in this rather simple case as well. We must know which architectural changes are induced by the physical counterpart of CS+US, and we must know how the modified system changes its activity, because the physical counterpart of the CS occurs in different dynamic contexts[81]. Today, we do not have this knowledge. Moreover, we cannot assert that we have the iteration of the same trajectory segment during training, and this observation can explain why it is difficult to obtain the same quantitative results in conditioning experiments.

We may observe difficulties in learning even when a subject follows the strategy of correctly repeating the same voluntary activity. The previous discussion may explain in particular why the history by which we reach the skill may become significant in learning to execute fluently a series of complex movements, such as in athletics or in playing a musical instrument[82]. Let the first part of the planned movement have as a counterpart some physical processes, and let these physical processes, in the conditions of the movement's occurrence, belong to a trajectory segment whose continuation does not contain the processes that are the counterpart of the continuation of the planned movement. Clearly the planned movement cannot occur in these conditions, and we can only try to change the conditions under which we execute the movement, or we can plan differently the details of the movement. We expect analogous situations in medical rehabilitation, particularly when rehabilitation significantly involves the plasticity of the nervous system.

The geometrical picture that we are using in this discussion is also useful to show a characteristic of this type of activity flow. The assumption that constrained activity will flow without interruptions implies that in defining this type of activity we will use a physical process that satisfies two conditions. It must mirror the properties of the projection of a single segment of the trajectories that describe the system dynamics in phase space[83]. The inverse map of the projection into the trajectory segment that projects onto it must be continuous: that is, no subsegment of the trajectory segment is orthogonal to the subspace onto which we project the trajectory segment. These requirements limit the possibility of obtaining a new item of this type by joining items of the same type, and we meet a further source of nonlinearities in the theory because the growth of this type of constraints cannot be reduced to a sequential composition of previous items.

When consider an activity that can flow with a certain number of interruptions, we meet weaker limitations in combining items of constrained activity to obtain a new item. The occurrence of two previous items can be separated by the occurrence of trajectory segments that are orthogonal to the subspaces of the projections that were used to define the two items.

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81. These two types of knowledge usually require different sets of experiments for repeatability reasons, and then the integration of the results.

82. In music we have a long history of "methods" to learn playing an instrument, or to gain a particular skill in playing it, and the sequence of the exercises is here essential.

83. Recall that we decided not to use the scheme of concurrent processes when we represent the dynamics of a physical system by means of trajectories in phase space.

However, we cannot use the temporal contiguity of two items for linking them in the new span of constrained activity, and we introduce a relation between the two items. In the geometrical picture that we are using, the constitutive activity of the relationship can be represented by the projection of a trajectory segment onto subspaces orthogonal to the previous ones. Nevertheless, this scheme is the simplest one, and we can think of more complex situations. For instance, thing that enter as first term in the relation can be a conscious memory, and we use a contiguous flow of activity to relate things that we think as being separated by a long interval of time.

We emphasize that we do not look on the constitutive activity of the first item and of the relation as being constitutive of the second item, although they occur in sequence at the occurrence of the second item. In our description, we thus can maintain a distinction between constraining and constrained spans of activity. This choice agrees with the observation that in this second type of constraints, constrained activity does not have to follow its constraining activity. We should come back to a deterministic physical description of the system activity to explain this difference between the two types of constrained activity, because I do not succeed in giving a methodological character to it.

The use of a relation between previously constrained spans of activity gives rise to new items that have strong relations with the previous ones. We thus can describe constrained activities as a system that grows during the life of an individual, and along the history of a group. In psychology, we usually identify this system with a conspicuous part of the experience and culture of the subjects.

Conditions under which a mental activity is performed are one source of this second type of constraints. For instance, the conditions (light, distance, etc.) under which a perceptive result has to be attained for having recognition of the objects, which we consider to be satisfactory. Subjects often signal the failure of these conditions by saying that the object appears with a certain shape or color, instead of saying that the object has certain shape or color.

The constraints on mental categorization are another typical example, and this fact is particularly evident in a scientific context where we usually bind the use of mental categories to the occurrence of specific technical procedures that involve physical things. For instance, we must use the techniques of geodesy and topography to assert that we measured again the distance between the same two points and their difference in height. 'Same' and 'other' are mental categories, and their occurrence uniquely requires that someone carry out the constitutive mental activity. However, we decide to use 'same' in this context only when certain technical procedures are well suited, otherwise we consider the categorization to be incorrect: that is, we recognize the category, and we consider as being incorrect its use in that context.

We do not always succeed in finding suitable technical procedures to which a categorization can be bound. An example is the assertion that a certain volume contains the same physical particles that it had at a past instant of time, because we do not succeed in identify the single particle in quantum mechanics. We then change the thing that we categorize as being the same. In this case, we develop theories where the datum is only the number of particles of a certain type that occupy a given volume at a certain instant of time. Then, it is matter of mathematical technology to use directly the datum, or to introduce equivalent mathematical transformations[84].

A compliant use of mental categories in scientific theories and in the description of scientific experiments, allows us to infer the occurrence of mental categorization in the particular context, although this method is not suitable to define mental categories. Indeed, the inference refers to mental categorization: that is, it refers to a mental category applied to particular things, and it depends on all the things that constitute the current situation. Since we infer that a mental categorization occurred from the occurrence of the technical procedures on which we constrained the categorization, we satisfy the repeatability requirement and we understand why it is important to find suitable technical procedures on which a categorization is constrained. Moreover, we always assume that the related technical procedures were correctly applied when we use the categories in a scientific context, and we expect the consequences that follow from their correct application. For these reasons, when we mention a category in a scientific context, we also refer to the procedure that constrains its use in the current context. Low awareness of this fact often caused bad philosophical statements.

We can suggest a way of defining this mental categorization[85] by referring again to the geometrical representation of a system dynamics as trajectories that do not intersect in phase space. Let *A* be a subspace of phase space, and let *s* be a projection of the trajectory segments. We can use *s* to define a mental categorization when the projections of *s* onto suitable subspaces of *A* coincide with the projections that we used for defining respectively the mental category and the thing categorized. In this scheme we usually have constraints on the timing of these projections, whici we omit for simplicity sake[86]. Clearly, a prerequisite is that *A* contains these subspaces. There is a large literature on mental categories and mental categorization in the Italian Operative School, and the reader can refer to the site that we mentioned in footnote at the beginning of the paper. We emphasize that, since we start from a projection *s*, many segments can share this projection, and they can belong to the same trajectory as well. We thus define mental categorization in such a way that it can occur again in the life of the same subject, and that it can occur identically in different subjects; but we have also the general consequences that we discussed above, when we plan to predict the occurrence of a mental categorization.

We often constrain mental categorization to characteristics of the things that we categorize. When we describe these characteristics by quantities that vary continuously, we usually constrain the mental categorization to certain threshold values of these quantities. We can consider the categorization as being a qualitative difference, and we can look at the thing that we categorized from two different points of view. We can consider it as a thing that was either categorized in a certain way or not, and so we have a qualitative difference. We can consider it only as having the characters to which we constrained the categorization, and from this point of view we have a continuous change in these characters. We thus have or do not have a qualitative difference depending on the viewpoint that we adopted, but we cannot expect that the thing will have this qualitative difference before the mental categorization. Let us, for instance, describe a characteristic of a physical body by its temperature. Due to the way in which we define and measure the temperature, we usually think of it as varying continuously. Let us now categorize this body as being cold. After categorization, we can think of the body as having the further qualitative character of being cold. Before we categorized it in this way,

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84. A common strategy has two steps. We firstly write the mathematical relations that should hold among physical particles that can be distinguished. In a second step we impose the conditions that must be satisfied by the mathematical description when we exchange two or more particles. In such a kind of situation it would be a good policy to avoid the use of terms such as ‘exchange force’ or ‘exchange interaction’, which may be misleading.

85. In other papers this type of mental activity was designated as “applied mental category“, or as applying mental categories.

86. We note that this scheme is substantially equal to the scheme that we proposed for relating two spans of constrained activity. We assume that the constitutive activity of mentally relating two things is mental categorization.

it was neither cold, nor hot. We emphasize that we can say that the body had a certain temperature only if we look on temperature as being a character that belongs to the definition of physical body: for instance, when we think of temperature as the average kinetic energy of the atoms or molecules, like in statistical mechanics of gases.

The characterization of a behavior as being intelligent shows another example of such a situation. Biology offers many examples, and one is the remarkable versatility in energy metabolism that is exhibited by purple photosynthetic bacteria. Many of these organisms can obtain cellular energy from light, from inorganic compounds, and even from organic compounds. The modes of producing cellular energy depend on the chemical and physical conditions of the environment, and they are so regulated to prevent unnecessary biosyntheses, like photosynthesis, which consumes large quantities of energy[87]. As second example, we can cite the behavior of the ciliate protozoan *Didinium*[88]. It has a globular body, about 150 microns in diameter, encircled by two fringes of cilia; its front end is flattened except for a single protrusion rather similar to a snout. The synchronous beating of the cilia allows the *Didinium* to swim in the water at high speed. When it encounters a suitable prey, that usually is another type of protozoan, it releases several small paralyzing darts from its snout region; and then it attaches to and devours the prey by contracting its plasma membrane to engulf the other cell. The prey has often the same dimensions of the *Didinium* cell. We can explain most of this complex behavior by physical processes that take place in the cytoskeletral structures lying just beneath the plasma membrane. We intentionally choose two biological examples that involve single cell organisms, because they clearly show that in biology no nervous system, or neural network, is necessary to have an adaptive behavior that might be considered as being an intelligent behavior.

The examples confirm that categorization introduces a distinction among things, in our case between intelligent and not intelligent things, and that the distinction depends on the categorization and its constraints (that is, on our cultural schemes) because we can easily find continuity when we look at the conditions on which the categorization is constrained. Cerebral death offers another example of such situation, and in this case we link deep practical consequences to the mental categorization.

The consequences that we expect from the occurrence of certain facts are a further great source of constraints. We have constraints of this type when we assume that certain objects have a defined role in certain processes, or that some facts shall follow from the occurrence of other facts. Since we do not necessarily require that the facts will be temporally contiguous, we have constraints of the second type. Few examples may better clarify the type of facts that we are referring to. We assume as paradigm that fire has the role of subject in an activity, burning, which transforms wood into ash. When the occurrence of the transformation and of the result is thought to be independent of our mental activity like in our case, we must check by observations and experiments whether the occurrence of the facts follows our assumptions. If our assumptions are not fulfilled, we often will explain the reasons, and in this case we use

87. The example of the *Rhodobacter capsulatus*, whose adaptive behavior was reviewed in C.E. Bauer and T.H. Bird, "Regulatory circuits controlling photosynthesis gene expression", *Cell*, 1996, 85, pp.5-8, shows many aspects of the physical explanation of this adaptive behavior, with particular emphasis on the system that regulates the expression of photosystem genes in response to alterations of the environment. The different environmental conditions induce the activation of different transmembrane proteins, and through different chains of chemical reactions they induce a different rate of synthesis of the proteins that are involved in the three modes of producing cellular energy. We will outline that a change in proportions of the same elements gives here a physical explanation of the adaptive behavior.

88. See B. Alberts, D. Bray, J. Lewis, M. Raff, K. Roberts and J.D. Watson, *Molecular biology of the Cell*, *cit*., pp. 24-25.

our assumptions as paradigm.

We have a further example when we assume that a subject acts to accomplish a certain result, and this example shows a very important effect of applying chains of constraints. Since we consider a subject's activity as being a purposeful activity, we are constrained to consider the activity as a mean to attain the proposed aim. When different activities bring to the same proposed result, other constraints induce us to consider these activities as being equivalent, from the viewpoint of attaining the proposed aim, although they can be thought of as being different from other points of view. Since this result follows from mental categorization, we always can concern ourselves with the differences between the activities that achieve the same aim. For instance, we can consider them as being a better or worst way of attaining the proposed aim, or we can explain why the subjects use different means to achieve the same aim. Note that low awareness of this character of mental categorization may lead to a rather naive philosophical realism.

We see that mental categorization often constrains a next, different mental categorization. When we discussed conscious memories, we met an example of this type of constraints. Since the subjects categorize a mental activity as a repetition of one that occurred to them in the past, they consequently consider the two activities as being equal. If a subsequent check shows differences, the subjects usually explain them, and in this case they use as a paradigm what we previously described as constraint.

Looking on a movement as being a voluntary movement offers another significant example of these constraints. In a voluntary movement, we consider the person or the animal that moves as being the subject of the movement. We will contradict ourselves if, at the same time, we would consider the movement as being made by something else. In fact, we would not consider as being voluntary a movement that we consider as being induced, for instance, by electrically stimulating suitable parts of the nervous system. Our cultural scheme is more complex, because it considers that a person or an animal can be induced to do an activity in many ways, and it correlates the degree of freedom that is assigned to the acting subject, with the strength of the constraints that we think to act on the subject. However, this more sophisticated scheme follows from a further assumption: we consider the subject of psychology, also as being the physical system of biology.

Constancy in perception

Perception psychology offers good examples of situations in which subjects report the same perceptive result although physical stimulation is different. For instance, the subjects usually report that they see their hands as having the same size at different distances, though the visual angle is very different, and the extension of the stimulated region in the retina as well. We see objects of the same color through a great range of light colors and intensities. We can thus interpret constancy in perception as examples of constrained mental activity. Many conditions, however, influence the occurrence and the strength of the constancy phenomena in visual perception. Significantly, size constancy of our hands is more evident when distances are in the range of reaching and grasping, and it fails when we look at a photo of the two hands placed at different distances. The constancy of colors is more evident when it involves objects whose color is well known to subjects. Furthermore, constancies are stronger in the adult life.

Let us consider in a physical description the two sets of processes that are involved in the previous description of constancy phenomena: that is, the set of physical processes that are

Figure 2

the counterpart of environment actions in the psychological description, and the set of processes that we used to define the constancy content in the psychological description. Geometrically we cannot think of these two sets of physical processes as being represented by trajectory segments in phase space, otherwise we would have a bijective function between the two orders of facts: that is, no constancy of the type described above. We must instead think of them as being represented by projections, onto suitable subspaces of phase space, of trajectory segments. The process too, that we consider as being the counterpart of environment actions in constancy is thus represented geometrically by a projection. Many trajectory segments can share this projection, and we cannot assume the projection to determine the following activity of the system. Constancies cannot thus have their unique cause in the environment actions, because this program is contradictory.

We also observe phenomena that in a certain way are the inverse of constancy, when we observe that we can correlate a different mental activity to environment actions in which the part remains equal that we assume as being sufficient to induce the different mental activities. Classical figure-ground alternations are good, controlled examples of these situations: for instance, the well-known Rubin figure-ground alternation examples, where we sometimes see a pair of faces, and sometimes a black vase (Figure 2 left), or where we alternatively lose as face the left or the right part of the figure (Figure 2 right)[89]. The physical situation that we use as stimulus remains equal in both the alternatives.

We also have experimental situations in which environment actions can support different perceptive or mental activity, although the physical situation that we use as stimulus remains equal in the two sessions. For instance, we may designate the pattern in Figure 3 as a line, or as an angle, and we can accept that someone still talks of a black pigment on the white paper of a page. In the framework of the Italian Operative School, this example was frequently used as a didactic tool to make a person aware of the role of mental activity. In particular, it was used to break the idea that we can set a one-to-one link between a physical description of the

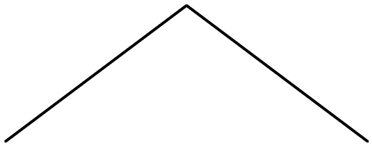

Figure 3

89. The two figures are taken from E. Rubin, *Visuell Wahrgenommene Figuren*, Kopenhagen, 1921.

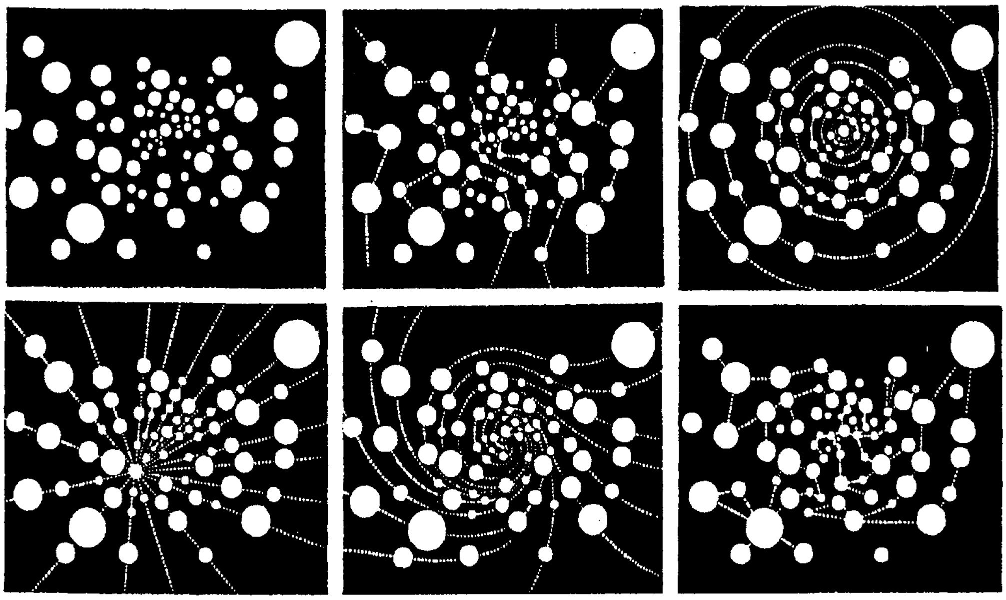

Figure 4

situation that is used as stimulus, and the occurrence of a mental fact, or of a linguistic behavior[90]. In this didactic context, a linguistic designation of the different alternatives was frequently used as constraint to induce different mental activities.

Perspective is another situation of this type, because a two-dimensional pattern leads us to perceive the room and the objects represented as being three-dimensional. This habit is today very strong, and we usually cannot escape it, because camera images, particularly the images that we see on television, are very frequently linear perspectives[91]. Nevertheless, we can see a perspective pattern as being two-dimensional, for instance when we are drawing it as an application of geometry, and we have again the possibility to think of the pattern as pigment on its support: paper, canvas, table, or wall. These alternatives may require further environment actions, for instance a particular context, or a previous activity, which acts as a sufficiently strong constraint to direct the activity in a particular direction.

The perceptive situations illustrated in Figure 4[92] show good examples of such an intermediate activity because they are induced by further environment actions. We can easily verify that the white circles have the same diameter and the same reciprocal positions in all the six figures. The further lines suggest different perceptual organization of the figure, and they seem to be quite necessary when we wish to obtain perceptual organizations that we can categorize as being ordered situations. In these cases, we can reasonably assume that the actions of the environment are different, because the further lines assume great importance.

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90. When the subjects consider the figure as a line or as an angle, we found some differences also in the movement of the eyeballs; see R. Beltrame, A. Berbenni, and G. Galassi, "Contribution to the studies of the movements of the eyeballs during optical perception by means of high speed motion picture photography", *Proceedings of the 7th International Congress on High-speed Photography*, edited by O. Helwich, Zurich, 1965, pp. 257-64.

91. A possible genesis of the linear perspective considered as a mental habit was discussed in R.Beltrame, *The Renaissance perspective. Birth of a cognitive fact*, Quaderni di *Methodologia*, 3, Roma 1996, 120 pp. A revision of this paper appeared as CNUCE Report C97-24, last revision Nov. 1998, (all in Italian).

92. The figures were prepared by P. Parini for the exhibition "Mind and Image", Gallery of Modern Art, Bologna, 1978.

Therefore, equality can result only by a comparison of the figures according to a different kind of criteria: the diameter of the circles and their reciprocal positions.

Mental categorization still participates to this aspect because it plays an important role in characterizing mental attitudes. We can define a mental attitude as a particular way of operating, which can be characterized by the occurrence of certain mental constructions, usually mental categories, or by the frequency of their occurrence[93]. With this type of definition, we can distinguish several mental attitudes, and not only those that historically received a designation, like, for instance, esthetical or ethical attitudes. However, we can think of many facts mentioned in this section as being a constraint on mental activity, and so as part of the dynamics of our systems.

The dynamics of constraints

As we have seen, we can speak of constraints that can be described as combination of other spans of constrained activity that may be not contiguous in time. We often speak of these constraints as part of the subjects' experience and culture, so that they can show a fast evolution, at least in certain periods of the subject's life. Communication and reasoning become ways to set up constraints, and we also can see culture as a system of constraints to mental activity. We enter here the enormous field of the development of mental activity in a single individual and in groups of individuals, a field of which we can offer only few elementary examples.

Let us have a color difference that we localize in our surrounding space, and that we think of as individuating a physical object. Adult persons usually think that there is also a tactile difference in the same place, and they expect to find it. For instance, we think of our hand reaching the place, and the change in tactile perception[94]. Conversely, we do not expect to find a tactile difference where we do not perceive visual differences. In fact, it is mandatory to mark clearly the presence of a glass door that is made of a single, transparent sheet; otherwise someone will walk into it. In this case, we can indicate the directions along which the paradigm grows. Very early in our life we get accustomed to adding a spatial localization to the color differences that we perceive in our visual field, and to think that we shall also find a tactile difference in the same place. This pattern of activities is learned as part of the coordination of the visual, motor, and tactile activity that we need to hold an object. Nevertheless, this simple paradigm becomes conditioned by other elements when our experience grows. For instance we do not add a tactile difference when the color differences involve something that we thought of as a plane figure, like a book illustration, and so on.

We can find another elementary example in a classical experiment on psychology of visual perception. In mono-ocular vision, and by suitably masking the context, the subjects report seeing experimental situations, like that in Figure 5, as being like that of the photograph in Figure 6[95]. So, they interpret the mutual positions of the objects as being in an order that is different from the order illustrated in Figure 5. We can explain this result by assuming that

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[93] A certain number of mental attitudes were defined following the approach outlined here in S. Ceccato, "A Model of the Mind", *cit.*; and in R. Beltrame, "Perceptive Operations", *cit.*

[94] We can obtain illusive effects by synchronizing visual, hearing, tactile, and smell stimuli according to the patterns that a person expects, although they arise from different physical situations than the usual ones. This is the leading idea of virtual reality; where the term 'virtual' highlights that the sources of the stimuli are different from those we assumed as a paradigm for these stimulations.

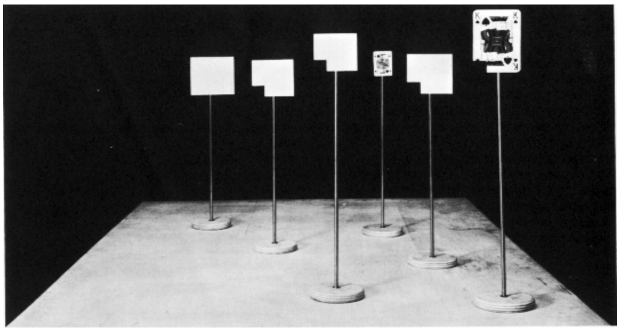

Figure 5

subjects think that all the squares and the cards are complete: that is, without the cuts that we see in Figure 5. Furthermore, we can explain that the subjects do not think of the situation as that illustrated in Figure 5, although it might be possible, because it is quite improbable on the basis of their experience. After thinking of the objects in the mutual positions illustrated in Figure 6, the subjects see them as having a size that depends on these mutual positions, and on the visual angle from which they see each object. This angle, indeed, settles a relation between the size and the distance of the object, and this relation describes a constraint on mental activity that results from learning[96]. In the conditions of the experiment, it plays the same role.

A television screen is a two-dimensional surface, but we usually see the rooms and the objects that are presented on it as being three-dimensional. The camera lens gives a result that is usually very near to a representation of the room and the objects on a plane perpendicular to the optical axis of the lens, and that follows the rules of linear perspective. Since the Italian Renaissance, we learned to see things that are represented in linear perspective as

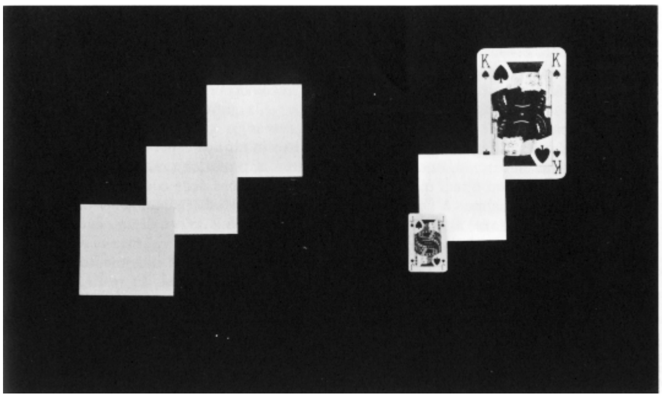

Figure 6

95. For experiments of this type see: J.J. Gibson, *Perception of the visual world*, Boston, Hampton, 1950; W.H. Ittelson, "Size as a cue to distance: static localization", *American Journal of Psychology*, 64, 1951, pp. 54-67; W.H. Ittelson, "The constancies in perceptual theory", *Psych. Rev.*, 58, 1951, pp. 285-294; A. Dinnerstein, W. Epstein, "The influence of assumed size on apparent distance", *American Journal of Psychology*, 76, 1963, pp. 257-265; L. Ancona, *The dynamics of the perception*, Mondadori, Milano, 1970, pp. 53-70 (in Italian), from which the figures were taken; J.E. Hochberg, *Perception*, 2nd Ed., New York, Prentice-Hall, 1978.

96. A good review of experimental results can be found in A. Yonas, C.E. Granrud, "The development of sensitivity to kinetic, binocular and pictorial depth information in human infants", in D. Ingle, D. Lee, M. Jeannerod (Eds.), *Brain Mechanisms and Spatial Vision*, Amsterdam, Martinus Nijhoff Press, 1984.

being three-dimensional; and the great diffusion, in our time, of images that are produced in this way by optical systems confirms this habit.

Furthermore, in watching television we also became accustomed to identify the position of the camera with our observation point, and the axis of the lens with the principal axis of the vision field. These assumptions, and the movements of the camera when filming, reinforce the tendency to think of the things represented as being three-dimensional. Note that we experience effects that are similar to stereokinetic ones. The television screen, the photograph of Figure 6, and other similar situations are good examples of a common situation: an acquired habit leads us to do a mental activity with a higher probability than other possible ones, because, for instance, we usually see a perspective as a two-dimensional pattern when we are drawing it.

In the case of a television screen we have an immediate example of the increasing sophistication of the constraints' scheme. We agree to add a tactile difference where we located a visual difference, but constraints are now different. We have constraints that involve the objects represented on the screen, and constraints that refer to objects of the environment of our body, like the television set. So, when two represented objects collide, we expect to see the usual consequences of a collision between two physical objects. However, we do not move from our chair when a car is represented on the screen as coming up to us. Such behavior requires a good level of cultural sophistication. The world of our experiences and of our knowledge must be applied to the situations that are represented on the screen; but, at the same time, we must expect that the represented actions have very different consequences on us than on the represented objects.

In other cases we observe a new, subtle use of existing constraints. Brunelleschi in his first experiment, which marks the beginning of modern perspective, used the acquired habits of his contemporaries to obtain that the observer gives a three-dimensional organization to the plane perspective of his drawing. He used the reflection on a flat mirror, and a very usual sight: the Baptistery as it is seen when a person comes out from the central door of the Florence cathedral[97]. These choices eliminate the ambiguity that is implicit in passing from the perspec-

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97. The first Brunelleschi's experiment was described in this way by his biographer: "He first demonstrates his system of perspective on a small panel about half a *braccio* square. He made a representation of the exterior of San Giovanni in Florence, encompassing as much of that temple as can be seen at a glance from the outside. In order to paint it, it seems that he stationed himself some three *braccia* inside the central port of Santa Maria del Fiore. He painted it with such care and delicacy and with such great precision in the black and white colors of the marble that no miniaturist could have done it better. In the foreground he painted that part of the piazza encompassed by the eye, that is to say, from the side facing the Miseriocordia up to the arch and corner of the sheep, and from the side with the column of the miracle of St. Zenobius up to the corner of the straw, and all that is seen in that area for some distance. And he placed burnished silver where the sky had to be represented, that is to say, where the buildings of the painting were free in the air, so that the real air and atmosphere were reflected in it, and thus clouds seen in the silver are carried along by the wind as it blows. Since in such a painting it is necessary that the painter postulate beforehand a single point from which his painting must be viewed, taking into account the length and width of the sides as well as the distance, in order that no error would be made in looking at it (since any point outside of that single point would change the shapes to the eye), he made a hole in the painted panel at that point in the temple of San Giovanni which is directly opposite the eye of anyone stationed inside the central portal of Santa Maria del Fiore. for the purpose of painting it. The hole was as tiny as a lentil bean on the painted side and it widened conically like a woman's straw hat to about the circumference of a ducat, or a bit more, on the reverse side. He required that whoever wanted to look at it place his eye on the reverse side where the hole was large, and while bringing the hole up to his eye with one hand, to hold a flat mirror with the other hand in such a way that the painting would be reflected in it. The mirror was extended by the other hand a distance that more or less approximated in small *braccia* the distance in regular *braccia* from the place he appears to have been when he painted it up to the church of San Giovanni. With the aforementioned elements of the burnished silver, the piazza, the viewpoint, etc., the spectator felt he saw the actual scene when he looked at the painting. I have had it in my hands and seen it many times in my days and can testify to it." H. Saalman, *The life of Brunelleschi by Antonio di Tuccio Manetti*, English translation of the Italian text by C. Enggass, The Pennsylvania State University Press, 1970, pp. 42-44.

tive painting to the place and the size of the objects that are represented in it. Since all the previous choices of Brunelleschi usually are not possible in painters' perspectives, suitable conventions about the characteristics of the represented things substituted them. The terrain is thought to be plain and horizontal, and it is usually an artifact: for instance, a rectangular place paved with square tiles. The vertical architectural elements - columns, or pillars - are thought to have the same height, and their sequences are thought to lie parallel or orthogonal to the plain of the perspective. In buildings, arches are thought to be semicircular, and so on. In conclusion, cultural facts lead again the subjects to perceive unambiguously as three-dimensional the objects and the places that are represented in the perspective plane[98].

These examples agree with the picture that we outlined in the previous sections. The second type of constraints still follows from learning, and we can continue to think of learning as an activity that brings the system in a volume of phase space where the constitutive activity of the learned facts has a higher probability to occur. How a particular system attains this result can be explained only by coming back to the physical description, and it is matter of experiment because the result does not have a methodological character.

The constrained activity often describes consequences that relate to subjects' body and to its interactions with the physical objects of the environment. These consequences have a strong impact on the subjects' actions and behavior because constrained activity usually concerns situations that occur with reasonable frequency, or that are critical for the subjects' survival. We have seen that, when we take the approach of psychology, constraints do not predict a sure occurrence of constrained activity, and the occurrence of the constrained activity may fail. When subjects are aware of the failure[99], they can:

- add new conditioning elements, and then use a more extended scheme of constraints: that is, the subjects require a richer and more articulated pattern of conditions to expect the occurrence of a fact[100];
- cease to consider a mental activity as being predictive of another, and consequently modify the scheme of constraints;
- decide not to pursue the mental activity that was just carried out, and substitute it with a mental activity from which the occurred consequences can follow (for instance, persons usually cease to consider something as being nearer to them than another thing, when further tests do not confirm the result of their perception, and they exchange the categorization of the two things).

We note that subjects assume as paradigm the relations between the constraining and the constrained things. When they modify their system of constraints, they consider themselves as dealing with relations among things, rather than with constraints on their mental activity. Subjects commonly use this viewpoint in deductive reasoning to predict facts of practical relevance: for instance, when they predict the consequences of their body interaction with other physical things. They use relations that occur frequently to accomplish this task, and they

98. These aspects of Renaissance perspective were discussed with more details in R. Beltrame, La prospettiva rinascimentale. Nascita di un fatto cognitivo *(The Renaissance perspective. Birth of a cognitive fact)*, *cit.* (in Italian).

99. Constraints are introduced in the description by theorists. We cannot thus identify them with a mental activity of the observed subject, otherwise we will have a *regressum ad infinitum*.

100. The increased number of conditions which an individual requires to be fulfilled in order to expect a fact can explain why aged and experienced individuals are more skeptical about the possibility to obtain a certain result. The increased number of conditions can contribute to increase their reaction time as well, because the individual will wait for the occurrence of more conditions before reaction starts.

change their paradigms only when very strong reasons force them. Furthermore, since from a contradiction we can deduce both a proposition and the opposite one, the presence of contradictions in this scheme would destroy the practical relevance of the deductions. Subjects thus require that this scheme will be free of contradictions, and this property of the scheme allows us to avoid ontological dualism between physical and mental things.

In the previous discussion of constraints, we mentioned, as main source, training procedures in which subjects are aware of obtaining a certain skill, or of increasing their knowledge system. However, actions of the environment on a biological system are not equally probable and independent. They have correlation. Gravitational field, for instance, induces a statistical predominance of vertical and horizontal lines[101]. Other shapes and orientations have an analogous predominance because of the technological procedures that we use to produce our artifacts. The sounds of the spoken language and the alphabet show other examples of correlation between actions of the environment, and we could continue our citations.

Correlation between environment actions influences the changes in the physical architecture of the biological systems, although we do not expect a simple relation between the two orders of facts. We mention this further source of constraints because it does not require that the subjects are aware of the results to be obtained, and neither of the activity from which the results originate.

We conclude our discussion with a general remark. If we define mental facts and activities as we proposed in this paper, the mental world acquires the character of a human construction, and, symmetrically, biology becomes a section of physics.

[101]. Fluids have a trend to arrange themselves in a way that locally gives rise to horizontal planes. Trees. on the other side, have a strong trend to grow vertically, because soil supports well vertical load, but it has a poor resistance to shear stress.

Appendix A - Further remarks on the physical description

As we have seen, in a physical description the system dynamics can be optimally represented by a theory in which we use systematically the cause-effect relation, and in which we choose the things to consider causes and effects so that a bijective function holds of the causes onto their effects. We recall that in this context the systems become the usual biological systems extended to a suitable part of their environment so that the enlarged physical system can be considered as being isolated. Environment actions thus become interactions between parts of the enlarged system, and we can completely predict the energy exchanges. Clearly, the total energy of the enlarged system is constant.

As we discussed, we can meet strong practical difficulties to fulfill these assumptions, mainly because of the severe difficulties to collect the information which is necessary to develop such an optimal theory for the biological systems; but we have no conceptual difficult. We will maintain these assumptions because they characterize an optimal physical theory in which the differences between the physical and the psychological description become very sharp, and the problems acquire a more clear formulation. Moreover, I think that they continue to hold in a more realistic approach, for instance the approach of statistical mechanics, since they have a methodological character. In this physical description the equations that describe the evolution of the system completely describe the dynamics of the system. So, they also describe the connection among the various segments of activity and the constraints on the activity flow. We emphasize that different configurations of the values of the observables, which characterize the state of the system, lead to a different evolution, because we assumed that a bijective function of the causes onto the related effects holds in the reference theory. We also emphasize that a bijection between the causes and their effect is a reasonable requirement, but it is typical of a theoretical attitude: that is, when our aim is to build a general theory. In practical activities we usually do not require this property. Different procedures thus can lead to the same result, and this possibility allows us to achieve an aim following more than one way. However, we have different side effects too, and these side effects are sometimes very significant also when we assume a practical attitude. In these cases, we have to manage them, as, for instance, in clinics.

We have no methodological problem when we succeed in individuating physical processes whose occurrence is equivalent to the action of conservative forces, because these processes allow us to calculate the global energy exchange, and so to predict the behavior of the system. Furthermore, because the conservative forces have potential, we need to know only the differences of the potentials between the initial and the final states of the system in every process to calculate the total energy that is exchanged during the process. Nonetheless we succeed to apply this strategy only in a very limited number of cases; in all the other cases we are involved in practical difficulties.

If we succeed in handling the occurrence of a physical process as being equivalent to the action of a force, but the force is not conservative, then it is usual to develop a theory in which the action of suitable conservative forces explains the action of the nonconservative force, and it also predicts the energy exchange due to the action of the nonconservative force. Usually there are many ways to realize this result, and here again the experiments decide the choice. The discriminating elements in this choice are the agreement with the predictions of the amount of energy exchanged by the nonconservative forces, and the values of other observables. We usually explain in this way friction, plastic strain, and the heat exchange phenomena that occur together with mechanical phenomena. In my opinion, this strategy supports the idea that the requirements stated above (forces are conservative, they do not depend explicitly on time, and we have to deal with an isolated system) are motivated methodological choices. However, we shall soon see that in many cases this strategy raises methodological problems as well.

The conservative forces that we introduce to explain the nonconservative force cannot have this force as resultant force, because this resultant force should be conservative, and this fact should contradict the hypothesis. For the same reason we cannot assume the nonconservative force to be only a component of the force obtained by the sum of the conservative forces, because this component too should be conservative. We can only assume the nonconservative force to be the statistical component of the action that the conservative forces exert in the direction of the nonconservative force. So, we can have zero as average value of the resultant of the conservative forces in other directions and during a certain interval of time, but the components of the conservative forces in these directions give a contribution to the energy that is exchanged in the same interval of time. However, the total energy exchange does not equal the energy exchange produced by the nonconservative force. The energy exchange becomes the sum of at least two parts that depend on the particular process. One of them is the work made by the nonconservative force, and the other is the further energy exchange that is necessary to equal the work made by the conservative forces that we conceptually introduce as methodological choice.

We have no problem when we know how to calculate the work that is made by the conservative forces. When instead we do not succeed in calculating the work made by conservative forces, we have to evaluate two energy exchanges that depend on the process by which the system passes from the same two states. We do not always succeed in finding two energy exchanges that depend on the particular process by which we pass from the same initial and final states, and whose sum depends only on these two states. In many cases we obtain only inequalities or bounds, but in the more favorable cases too, we need a greater number of parameters than in the case in which we know how to calculate the work made by the conservative forces, because now we have to characterize the process.

When the occurrence of two processes can be assumed to be equivalent to the action of the same conservative force, and when one process leads the system from the final to the initial state of the other process, the sequence of the two processes leaves the system in the same state, and the total energy that is exchanged in the sequence of the two processes is clearly zero. Because we are thinking of isolated systems, we can conclude that a process whose occurrence is equivalent to the action of a conservative force is completely reversible. When we do not succeed in describing the actions of our system in this way, we frequently have to deal with irreversible processes, because the energy exchange now depends on the particular process as well, and may happen that we do not succeed to induce a physical process that is the inverse of a given one. Furthermore, we often meet the severe difficulties of studying nonequilibrium phenomena[102], because we had to assume that the nonconservative

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102. See, for instance, W. Grandy Jr., *Foundations of Statistical Mechanics*, Vol. 1, *Equilibrium Theory*, Boston, 1986; and Vol. 2. *Nonequilibrium Phenomena*, Boston, 1988. The introductory chapters of the second volume outline very clearly the methodological bases of the approach to nonequilibrium phenomena.

forces are the statistical components of conservative forces, and their average over a suitable interval of time may significantly depend on this interval length, and we might not succeed in approximating our processes with a sequence of equilibrium states[103].

We emphasize that nonequilibrium situations are quite the rule in physical descriptions, because they occur in studying transient phenomena, and transient phenomena are the core of every dynamic. We have again to deal with nonequilibrium phenomena when the value of an observable depends on the interval of time during which we maintain the interaction between the system and the measure apparatus. In developing a theory of nonequilibrium phenomena we thus must decide how to interpret the measured values of the observables, and this problem is often crucial for the theory.

Finally, we may not succeed in decomposing a physical process into more simple processes whose occurrence is equivalent to the action of a single conservative force of our theory. In these conditions we again consider the occurrence of the process to be equivalent to the action of a suitable number of conservative forces, and we introduce new conservative forces in our theory. Then, starting from the results of the experiments, we must introduce suitable relations between the conservative forces, and their dependence on the parameters that characterize the process.

The differences that we stated between the approach of physics and the approach of psychology may be identified also when we will use a more realistic physical description, and when we will consider quantum mechanics aspects of the dynamics of the biological molecules. Nevertheless, I did not succeed in tracing them with sufficient clarity, when I tried to give a physical description of a biological system's behavior by following the approach of the statistical mechanics or of the continuum mechanics. I think that the conceptual scheme of the continuum mechanics[104] might be another good formal tool to describe biological systems, particularly the more complex ones. If we use a point of view that thinks of the system as being composed by discrete elements, then the biological systems would have an extremely high number of elements, thus it would become very difficult to manage, both conceptually and mathematically. Moreover, in these systems there is traffic of chemical molecules and ions of different sizes among the different parts of each cell, among the various cells, and among cells and their extra cellular matrix. The continuum with microstructure[105] probably would offer a better viewpoint, but some difficulties arise from our aim to introduce delay in the interaction. In continuum mechanics extension is atomic. Thus, we do not define point values of the observables, but distributions, and we have to introduce fields to describe the interaction[106]. These facts should force us to develop the system dynamics with instruments that are slightly different from those discussed in this paper. So, we prefer not to deal with this possibility here. Moreover, the probabilistic character of the predictions, which we obtain from certain theoretical approaches of physics, may mask the consequences of having defined mental things by using

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103. When we assume the system to be in an equilibrium state, we recall that a very strong methodological constraint is used, often implicitly. Feynman exposes it with the following words: "That every process must, in thermal equilibrium, be balanced by its exact opposite is called the *principle of detailed balancing*." R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman lectures on Physics*, *cit*., Vol. I-2, 42-5.

104. See C. Truesdell and R. Toupin, *The classical field theories*, Encyclopedia of Physics, Vol. III/1, Springer-Verlag, New York, 1960; C. Truesdell and W. Noll, *The non-linear field theories of mechanics*, Encyclopedia of Physics, Vol. III/3, Springer-Verlag, New York, 1965.

105. A good introduction is in G. Capriz, *Continua with microstructure*, Springer-Verlag, New York, 1984.

106. See for instance J. Glimm and A. Jaffe, *Quantum Physics. A functional integral point of view*, 2nd edition, Springer Verlag, 1987.

only part of the physical processes, and it thus can mask the different origin of the probabilistic character of the predictions. The similarity does not really go beyond the use of the same mathematical instruments to formalize a probabilistic approach.

We will outline a difference between the description of biological systems and the description of physical system whose theory is well assessed.

In a theory of a system like a crystalline solid, we obtain good results if we introduce the interaction of each element with only the few elements immediately surrounding it, and in which we think that this situation holds for all the elements of the system. Clearly, we have an exception for the elements at the boundary of the system with its environment. The interactions of these boundary elements determine a large part of the system's interaction with its environment; the other part being described as a further interaction of each element with an external field: that is, an interaction with a field that covers the system, and that we think of as being caused by other physical systems.

In biological systems, we find two phenomenological data that prevent us from assuming a short-range interaction as a general prototype of the interactions among the parts of the system. A piece of cat does not behave like a cat; instead, a reasonably small amount of sodium chloride behaves like sodium chloride. Therefore, the description of the interaction among the parts must have considerable differences in biological and in physical systems, and different theoretical models are required. If macroscopic parts lose the behavior in which we are interested when we isolated them from the surrounding ones, then we must add to the theory significant interactions among distant parts of the system. We still must add actions of the environment onto the system; because we again observe that, when an environment action ceases to act, this fact causes the loss of the behavior in which we are interested. This situation usually occurs at the level of cell: thus, it is almost the rule in biological systems.

Indeed, in biological systems we frequently observe the degeneration of the parts that become excluded, for any reason, from interacting with other parts of the system, or with the environment. In some conditions, this lack of interaction induces a change of function: for instance, when we observe a modification in the extension of the cerebral cortex areas that are interested in visual, auditory, and tactile activities as consequence of lesions or diseases that strongly reduced the visual or auditory function. In other conditions we can see, particularly in animals, a voluntary behavior to deprive oneself of a body part that has lost its functionality, for instance as a consequence of a lesion. Finally, if long-range interactions cease for a sufficient interval of time, then the system loses its interconnections and its stability, so that, at room temperature, it starts to decompose itself. It is common knowledge that we have to maintain the biological material at a reasonable low temperature to preserve its architecture, and to avoid its decomposition. A computer card, instead, has a reasonable stability at room temperature both when it is functioning in a computer, and when it does not function, for instance, because it is not powered[107].

Since in biological systems we must use long-range interactions, and since the system's stability requires an intense activity, we can expect that at least two ways of storing energy

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[107]. Note that we cannot define the analogous of free particle in mechanics, or of the void in field theory, when we del with biological systems: that is, we cannot define a situation in which we assume that no interaction is acting because a biological system is typically an open system. We have to define these things for the reference physical system that we outlined in a previous section, and which is an isolated system. However, we will not concern with this problem because we shall wander from the main topic of this paper, which refers to the methodological aspects of integrating the approaches of physics and of psychology.

will be significant. The most frequently mentioned way is the presence of molecules that participate in chemical reactions, and a release of energy is associated with them. ATP (adenosine 5'-triphosphate) is one of these molecules. The second way is the energy exchange between parts of the system such that the loss of energy is very low during the exchange. The mathematical prototype of this energy storage is the harmonic oscillator; but, in principle, every periodic process may be a good candidate to store energy in this way, and, significantly, biological systems show many periodical processes. We thus have two ways to think of the process of storing energy in our systems, and their dynamics is different.

Despite the strong necessity to consider interactions among remote parts of biological systems, we can study some aspects separately. A good example of this strategy is the theory of various types of the so-called receptors. We think of receptors as parts of the biological system that respond to environment actions only in a narrow band, and with a high gain. The main problem of receptors' theory is usually the correlation of the receptors' activity with the physical actions to which they can respond, and the very high specificity of the interaction allows us to study separately the many kinds of receptor, although the successive activity that they promote must be studied as we previously discussed. When we have environment actions that depend on previous actions of the system we are however forced to enlarge the study, and to include in the theory a suitable part of the environment. Actions that modify the subject's environment offer an example of this necessity. Actions onto the environment, which imply the activity of muscles, determine the actions of the environment on the sensory receptors, which follow from the previous surrounding modifications, and we must connect these activities in the theory. Situations of this type occur just for a simple displacement of an object, and they show other strong reasons for extending the system with which we have to deal in the theory. These extensions quickly lead to the isolated system that we proposed for the reference physical theory. In this enlarged system, we describe the previous actions through cause-effect relations between physical processes that occur in certain parts of the isolated system.

We further emphasize that, although we describe the interactions between physical things by conservative forces, these forces might not depend linearly on the independent variables of the interaction. For instance, we put two electrically charged bodies at a certain mutual distance to produce electrostatic forces. Experiments show that electrostatic force is conservative, but it depends on the product of the two charges, and on the inverse square of the mutual distance between the two charged bodies. In these conditions, non-linear equations describe the behavior of the charged bodies, and this conclusion continues to hold when we have a system composed by many parts, although we can linearly combine the conservative forces that describe the interactions between these parts. Other sources of non-linearity may arise when in the physical theory we do not succeed in describing interactions by means of conservative forces[108]. In these cases, energy exchanges also depend on the particular process that the system is doing, and a principle of superposition does not hold. Finally, we remember that, when we are dealing with physical systems that cannot be considered as being isolated, the principle of superposition requires linear boundary conditions to hold as well.

We will close this section by discussing some methodological consequences of using different elemental interactions for describing the dynamics of a physical system. We clearly meet the simplest situation when the interactions are mutually independent and the resulting interaction has the same properties of the component ones. However, we must satisfy two strong

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108. We will briefly discuss this type of situation in Appendix A - Further remarks on the physical description.

conditions to achieve this result:

- we must use a linear composition of the elemental interactions;
- we must be concerned with the same effect of the different interactions: for instance, their mechanical effects on the same body.

When we deal with different effects, we must check whether those properties of the compound subsist that were predicted by the current theory, and we must investigate whether the composition gives rise to new effects.

The classical theory of electromagnetic field offers a clear example of this state of facts. We take an electric charge, for instance a little sphere with a positive charge, and we move this sphere at a certain velocity. We may try to predict the resulting effect by combining two situations whose theory is well known, the electrostatics and the elementary mechanics. However, a new effect arises, because we obtain a magnetic field too. This further effect is a property that we cannot logically deduce by electrostatics and elementary mechanics, and its knowledge must be obtained by experiments on the situation that results from moving a mechanical body that carries an electrical charge. These experiments are also necessary to describe the dependence of these effects on other observables, and then to enlarge the theory. Finally, since the experimental situation is changed, we still need further experiments to check whether the relations among the observables continue to hold, which were predicted by classical mechanics and electrostatics: that is, by the theories of the two situations from which we started. Here, again, only experiments can decide the changes, and we know that the laws of classical mechanics and of electrostatics generally do not continue to hold, but we have to amend them so that they give results that agree with the experiments in the new range of conditions[109].

It is outside the scope of this paper to discuss how in physics a theory of the electromagnetic phenomena was obtained, which satisfies the methodological requirements stated in the previous sections[110], but this example shows a situation that occurs rather frequently. When we combine elementary situations of different types, we are using a non-linear law of composition, and we cannot predict the properties of the compound by deducing them logically from the properties of the components. In physics, as we mentioned above, we must use experiments to check whether the properties of the compound subsist, which are predicted by the current theory, and to investigate possible new properties of the compound. In mathematics, we usually define new objects, and we must deduce what is implicit in these new definitions. We have an example of this procedure when we think that a geometrical entity with the characters of a surface can be obtained by composing entities with the properties of a line. If the lines are straight lines, the surface is a plane, and we can define a new class of geometrical objects on the plane: the angles, whose properties we have to deduce from a new definition, because they could not be defined on a line.

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[109]. We know that, when the velocity of the charged sphere is relatively low, the surfaces having the same electrostatic potential can again be considered as spheres with the center on the moving charge; that is the same theory holds that we find when the charge is at rest. When the velocity is near the velocity of light in a very rarefied gas (the so-called void), these surfaces must be considered ellipsoids, Lorentz's transformations hold, and the mechanical momentum becomes a non-linear function of the velocity. More details can be found, for instance, in R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman lectures on Physics*, *cit*.,

[110]. Recall that the magnetic field generated by a moving electrical charge depends on the velocity of the moving electrical charge. So, in this formalization we have a first time derivative, and it is not invariant under time reflection. We know, however, that we can introduce a vector and a scalar potential, thus transforming the original formalization into a formalization which has the required properties, and which equally satisfies the experimental results. See, for instance, M. Kaku, *Quantum field theory. A modern introduction,* Oxford University Press, New York, 1993, Chap. 4.

The examples show a general fact. When two, or more, different physical processes occur together, we must test by means of experiments whether we observe new effects and changes in the effects that each process produces when it acts alone. A non-linear composition law thus implies that processes cannot be considered as being independent, and models become useless whose global properties and dynamics follow from statistics in which we assumed the elemental interactions to be independent, or equally probable. Even if we take an approach that uses the correlation between the observed events to explain and predict the behavior of our systems, we expect acceptable results only from models in which a strong correlation was introduced between the events, and we do not expect to obtain satisfactory predictions by a scheme of random, independent events. However, we shall have the related mathematical difficulties.

Furthermore, in the experiments we cannot deal with Boolean variables: for instance, the presence or absence of a chemical substance. In a linear dynamics, the dependence on one parameter does not alter the dependence on other parameters, and we can correctly study the dependence on one parameter by excluding the others. In a non-linear dynamics, this strategy may give worse results, because we can have a dependence on the product of two or more parameters. Then, by assigning a zero value to one parameter, we may mask the dependence on others.

The relation between dependent and independent variables depends on the values of the parameters that characterize each experiment. However, in a linear dynamics we know the dependence law by definition, in a non-linear dynamics, we instead must devise a suitable dependence law. This task requires a greater number of experiments than in the case of a linear dependence, because we must test with different combinations of the values of the parameters that characterize a single experiment. We often describe a situation in which we have non-linearity as a situation in which we have synergies. We emphasize that, when we assign a function to a single anatomical piece, or to a single chemical substance, e.g. a protein, we are implicitly considering the dynamics of our system to be linear with respect to the realization of that function, although the function may be promoted by a non-linear dynamics.